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A.1 Proof of the Cooperativeness Order

When each strategy is denoted by a finite automaton, we assume that an implementation error is made in the choice of an action in each state, and not in transition from the current state to the next. We also assume that the errors are independent and identically distributed between the players and across rounds. Denote by $\varepsilon \in [0, \frac{1}{2}]$ the probability of such an error. For the analytical comparison of cooperative levels, we assume that $\varepsilon$ is small. In some cases considered below, this implies that we treat $\varepsilon^2$ as negligible. In other cases, however, we need to consider the difference in the order of $\varepsilon^2$ and treat $\varepsilon^3$ as negligible. Let $p = (1 - \varepsilon)^2$, $q = \varepsilon(1 - \varepsilon)$ and $r = \varepsilon^2$. The normalized stage payoffs with implementation errors are given by

$$g_{CC} = p + q(1 + g - \ell), \quad g_{CD} = p(-\ell) + q + r(1 + g),$$

$$g_{CD} = p(1 + g) + q + r(-\ell), \quad g_{DD} = q(1 + g - \ell) + r,$$

where $g = 1$ and $\ell = 17/12 \approx 1.416$ in our implementation. Define

$$g = \begin{bmatrix} g_{CC} \\ g_{CD} \\ g_{DC} \\ g_{DD} \end{bmatrix}.$$

We consider a Markov process induced by a pair of the same strategy implemented with errors $\varepsilon$. Let $\Theta$ be the set of states of this Markov process. For each strategy that can be expressed as an $S$-state automaton, $\Theta$ can have up to $S \times S$ elements. The Markov process is defined over the set $\Delta \Theta$ of distributions over those states. Let $\omega^1 \in \Delta \Theta$ be the row vector representing the initial distribution and $A = (a_{st})_{s,t \in \Theta}$ be the transition matrix: $a_{st}$ is the probability that the next state is $t$ when the current state is $s$. The distribution $\omega^2$ over round 2 states is given by $\omega^2 = \omega^1 A$, and the distribution $\omega^t$ over round $t$ states is given by $\omega^t = \omega^1 A^{t-1}$. With the distribution $\omega$ over states, the expected stage payoff to a player is given by $\omega g$. In the case of the finite games, the average payoff over eight rounds can be computed as

$$\frac{1}{8} \sum_{t=1}^{8} \omega^t g = \frac{1}{8} \omega^1 (I + A^1 + \cdots + A^7) g. \quad (1)$$

\(^{73}\text{Hence, } \varepsilon = 1 - \beta \text{ for the parameter } \beta \text{ in SFEM.}\)
In the case of the indefinite games, the average discounted payoff can be computed as

\[(1 - \delta) \sum_{t=1}^{\infty} \omega^t \delta^{t-1} g = (1 - \delta) \omega^1 (I + \delta A^1 + \cdots + \delta^t A^t + \cdots) g \]

\[= (1 - \delta) \omega^1 (I - \delta A)^{-1} g, \quad (2)\]

where \(\delta = 7/8\) in our implementation. If we denote by \(v_{\theta}\) the average discounted payoff in the indefinite games along the Markov process with the initial state \(\theta\) (i.e., the initial distribution \(\omega^1\) places probability one on state \(\theta\)), and by \(v = (v_{\theta})_{\theta \in \Theta}\) the corresponding column vector, then (2) implies the recursive equation

\[v = (1 - \delta) (I - \delta A)^{-1} g \iff v = (1 - \delta) g + \delta A v. \quad (3)\]

### A.1.1 Indefinite games with small implementation errors

1. TFT and STFT: These strategies have two states 0 and 1. Both strategies play \(C\) in state 0, and \(D\) in state 1. Because the implementation errors occur independently between the two players, state transitions do not synchronize between them. Accordingly, the Markov process has four states \(\Theta = \{(0,0), (0,1), (1,0), (1,1)\}\). The initial distribution is \(\omega^1 = (1,0,0,0)\) if both play TFT and \(\omega^1 = (0,0,0,1)\) if both play STFT. We hence have \(v_{\text{TFT}} = v_{00}\) and \(v_{\text{STFT}} = v_{11}\). The transition matrix is given by

\[A = \begin{bmatrix} p & q & q & r \\ q & r & p & q \\ q & p & r & q \\ r & q & q & p \end{bmatrix}. \]

Ignoring the terms of order \(\epsilon^2\), we can write (3) as

\[
\begin{bmatrix} v_{00} \\ v_{01} \\ v_{10} \\ v_{11} \end{bmatrix} = (1 - \delta) \begin{bmatrix} g_{CC} \\ g_{CD} \\ g_{DC} \\ g_{DD} \end{bmatrix} + \delta \begin{bmatrix} 1 - 2\epsilon & \epsilon & \epsilon & 0 \\ \epsilon & 0 & 1 - 2\epsilon & \epsilon \\ \epsilon & 1 - 2\epsilon & 0 & \epsilon \\ 0 & \epsilon & \epsilon & 1 - 2\epsilon \end{bmatrix} \begin{bmatrix} v_{00} \\ v_{01} \\ v_{10} \\ v_{11} \end{bmatrix}. \quad (4)
\]

It follows from the second and third rows of (4) that

\[
\begin{bmatrix} v_{01} \\ v_{10} \end{bmatrix} = (1 - \delta) \begin{bmatrix} g_{CD} \\ g_{DC} \end{bmatrix} + \delta \begin{bmatrix} v_{10} \\ v_{01} \end{bmatrix} + \delta \epsilon \begin{bmatrix} v_{00} + v_{11} - 2v_{10} \\ v_{00} + v_{11} - 2v_{01} \end{bmatrix} = (1 - \delta) \begin{bmatrix} g_{CD} \\ g_{DC} \end{bmatrix} + \delta \begin{bmatrix} v_{10} \\ v_{01} \end{bmatrix} + O(\epsilon),
\]

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where $O(\varepsilon)$ is the term of order $\varepsilon$. Hence,

$$
\begin{bmatrix}
1 & -\delta \\
-\delta & 1
\end{bmatrix}
\begin{bmatrix}
v_{01} \\
v_{10}
\end{bmatrix}
= (1-\delta)
\begin{bmatrix}
g_{CD} \\
g_{DC}
\end{bmatrix}
+ O(\varepsilon).
$$

Solving this, we get

$$
\begin{bmatrix}
v_{01} \\
v_{10}
\end{bmatrix}
= \frac{1}{1+\delta}
\begin{bmatrix}
1 & \delta \\
\delta & 1
\end{bmatrix}
\begin{bmatrix}
g_{CD} \\
g_{DC}
\end{bmatrix}
+ O(\varepsilon).
$$

Substituting this into the first and fourth rows of (4), we obtain

$$
\begin{bmatrix}
v_{00} \\
v_{11}
\end{bmatrix}
= (1-\delta)
\begin{bmatrix}
g_{CC} \\
g_{DD}
\end{bmatrix}
+ \delta(1-2\varepsilon)
\begin{bmatrix}
v_{01} \\
v_{10}
\end{bmatrix}
+ \delta\varepsilon
\begin{bmatrix}
1 & \delta \\
\delta & 1
\end{bmatrix}
\begin{bmatrix}
g_{CD} + g_{DC} \\
g_{CD} + g_{DC}
\end{bmatrix}
+ O(\varepsilon^2).
$$

This can be rewritten as

$$
\begin{bmatrix}
1-\delta + 2\delta\varepsilon \\
0
\end{bmatrix}
\begin{bmatrix}
v_{00} \\
v_{11}
\end{bmatrix}
= (1-\delta)
\begin{bmatrix}
g_{CC} \\
g_{DD}
\end{bmatrix}
+ \delta\varepsilon
\begin{bmatrix}
g_{CD} + g_{DC} \\
g_{CD} + g_{DC}
\end{bmatrix}
+ O(\varepsilon^2).
$$

Ignoring the terms involving $\varepsilon^2$, we hence obtain

$$
\begin{bmatrix}
v_{TFT} \\
v_{STFT}
\end{bmatrix}
= \begin{bmatrix}
v_{00} \\
v_{11}
\end{bmatrix}
= \frac{1}{1-\delta + 2\delta\varepsilon}
\begin{bmatrix}
(1-\delta)g_{CC} + \delta\varepsilon(g_{CD} + g_{DC}) \\
(1-\delta)g_{DD} + \delta\varepsilon(g_{CD} + g_{DC})
\end{bmatrix}.
$$

2. Grim: The strategy has two states 0 and 1 where it chooses $C$ and $D$, respectively. State transitions are synchronized between the two players when they both play Grim so that the Markov process has only two states $\Theta = \{(0,0),(1,1)\}$. We have $\omega^1 = (1,0)$ so that $v_{\text{Grim}}^{\text{Grim}} = v_{00}$. The transition matrix is given by

$$
A = \begin{bmatrix}
p & 1-p \\
0 & 1
\end{bmatrix}.
$$

Ignoring the terms of order $\varepsilon^2$, we can write (3) as

$$
\begin{bmatrix}
v_{00} \\
v_{11}
\end{bmatrix}
= (1-\delta)
\begin{bmatrix}
g_{CC} \\
g_{DD}
\end{bmatrix}
+ \delta
\begin{bmatrix}
1-2\varepsilon & 2\varepsilon \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
v_{00} \\
v_{11}
\end{bmatrix}
$$

This yields

$$
v_{\text{Grim}}^{\text{Grim}} = v_{00} = \frac{(1-\delta)g_{CC} + 2\delta\varepsilon g_{DD}}{1-\delta + 2\delta\varepsilon}.
$$
3. Grim2: The strategy has three states 0, 1 and 2, where it chooses $C$, $C$, and $D$, respectively. State transitions are synchronized between the two players so that the Markov process has three states $\Theta = \{(0,0), (1,1), (2,2)\}$. We have $\omega^1 = (1,0,0)$ so that $v^{\text{Grim2}} = v_{00}$. The transition matrix is given by

$$ A = \begin{bmatrix} p & 1-p & 0 \\ p & 0 & 1-p \\ 0 & 0 & 1 \end{bmatrix}. $$

We can write (3) as

$$ \begin{bmatrix} v_{00} \\ v_{11} \\ v_{22} \end{bmatrix} = (1-\delta) \begin{bmatrix} g_{CC} \\ g_{CC} \\ g_{DD} \end{bmatrix} + \delta \begin{bmatrix} (1-\varepsilon)^2 & \varepsilon(2-\varepsilon) & 0 \\ (1-\varepsilon)^2 & 0 & \varepsilon(2-\varepsilon) \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_{00} \\ v_{11} \\ v_{22} \end{bmatrix}. $$

Solving this, we obtain

$$ v^{\text{Grim2}} = v_{00} = \frac{(1-\delta)\{1+\delta\varepsilon(2-\varepsilon)\}g_{CC} + 4\delta^2\varepsilon^2g_{DD}}{(1-\delta)\{1+\delta\varepsilon(2-\varepsilon)\} + 4\delta^2\varepsilon^2}. $$

4. TF2T: The strategy has three states 0, 1 and 2, where the action choices are $C$, $C$, and $D$, respectively. Since state transitions are not synchronized, the Markov process has $3 \times 3 = 9$ states $\Theta = \{(0,0), \ldots, (2,2)\}$. We have $\omega^1 = (1,0,\ldots,0)$ so that $v^{\text{TF2T}} = v_{00}$. The transition matrix is given by

$$ A = \begin{bmatrix} p & q & 0 & q & r & 0 & 0 & 0 & 0 \\ p & 0 & q & q & 0 & r & 0 & 0 & 0 \\ q & 0 & r & p & 0 & q & 0 & 0 & 0 \\ p & q & 0 & 0 & 0 & 0 & q & r & 0 \\ p & 0 & q & 0 & 0 & 0 & q & 0 & r \\ q & 0 & r & 0 & 0 & 0 & p & 0 & q \\ q & p & 0 & 0 & 0 & r & q & 0 & 0 \\ q & 0 & p & 0 & 0 & 0 & r & 0 & q \\ r & 0 & q & 0 & 0 & 0 & q & 0 & p \end{bmatrix}. $$

Using (3), we have

$$ \begin{align*}
\nu_{11} &= (1-\delta)g_{CC} + \delta v_{00} + O(\varepsilon) \\
\nu_{02} &= (1-\delta)g_{CD} + \delta v_{10} + O(\varepsilon) \\
\nu_{20} &= (1-\delta)g_{DC} + \delta v_{01} + O(\varepsilon). \quad (5)
\end{align*} $$
Substituting these into the recursive equations for $v_{01}$ and $v_{10}$, we obtain

$$
\begin{bmatrix} v_{01} \\ v_{10} \end{bmatrix} = (1 - \delta) \begin{bmatrix} g_{CC} \\ g_{CC} \end{bmatrix} + \delta(1 - 2\varepsilon) \begin{bmatrix} v_{00} \\ v_{00} \end{bmatrix} + \delta(1 - \delta)\varepsilon \begin{bmatrix} g_{CD} \\ g_{DC} \end{bmatrix} + \delta \varepsilon \begin{bmatrix} 0 \\ 1 + \delta \\ 1 + \delta \\ 0 \end{bmatrix} \begin{bmatrix} v_{01} \\ v_{10} \end{bmatrix} + O(\varepsilon^2),
$$

which yields

$$
\begin{bmatrix} v_{01} \\ v_{10} \end{bmatrix} = \frac{1 - \delta}{1 - \delta^2\varepsilon^2(1 + \delta)^2} \begin{bmatrix} 1 & \delta\varepsilon(1 + \delta) \\ \delta\varepsilon(1 + \delta) & 1 \end{bmatrix} \begin{bmatrix} g_{CC} + \delta\varepsilon g_{CD} \\ g_{CC} + \delta\varepsilon g_{DC} \end{bmatrix} + \frac{\delta(1 - 2\varepsilon)v_{00}}{1 - \delta^2\varepsilon^2(1 + \delta)^2} \begin{bmatrix} 1 & \delta\varepsilon(1 + \delta) \\ \delta\varepsilon(1 + \delta) & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + O(\varepsilon^2).
$$

It then follows that

$$
v_{01} + v_{10} = \frac{(1 - \delta)\{1 + \delta\varepsilon(1 + \delta)\}}{1 - \delta^2\varepsilon^2(1 + \delta)^2} \{2g_{CC} + \delta\varepsilon(g_{CD} + g_{DC})\}
$$

$$
+ \frac{2\delta(1 - 2\varepsilon)\{1 + \delta\varepsilon(1 + \delta)\}}{1 - \delta^2\varepsilon^2(1 + \delta)^2} v_{00} + O(\varepsilon^2)
$$

$$
= \frac{(1 - \delta)}{1 - \delta\varepsilon(1 + \delta)} \{2g_{CC} + \delta\varepsilon(g_{CD} + g_{DC})\}
$$

$$
+ \frac{2\delta(1 - 2\varepsilon)}{1 - \delta\varepsilon(1 + \delta)} v_{00} + O(\varepsilon^2). \quad (6)
$$

On the other hand, the recursive equation for $v_{00}$ yields

$$
v_{00} = \frac{(1 - \delta)g_{CC} + \delta\varepsilon(1 - \varepsilon)(v_{01} + v_{10}) + \delta^2\varepsilon^2 v_{11}}{1 - \delta(1 - \varepsilon)^2}. \quad (7)
$$

Substituting (5) and (6) into (7) and ignoring the terms of order $\varepsilon^3$, we obtain

$$
v_{TF2T} = v_{00} = \frac{\{1 + \delta(1 - \delta)\varepsilon - \delta^2\varepsilon^2\}g_{CC} + \delta^2\varepsilon^2(g_{CD} + g_{DC})}{1 + \delta(1 - \delta)\varepsilon - \delta(1 - 2\delta)\varepsilon^2}.
$$

As for the strategies AC, AD, and T6-T8, it can be readily verified that their cooperativeness is given as follows.

5. AD: $v^{AD} = g_{DD}$. 
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6. AC: \( v^{AC} = g_{CC} \).

7. T8: \( v^{T8} = (1 - \delta^7) g_{CC} + \delta^7 g_{DD} + O(\varepsilon) \).

8. T7: \( v^{T7} = (1 - \delta^6) g_{CC} + \delta^6 g_{DD} + O(\varepsilon) \).

9. T6: \( v^{T6} = (1 - \delta^5) g_{CC} + \delta^5 g_{DD} + O(\varepsilon) \).

Combining the above cases, we can rank the ten strategies from the least cooperative to the most cooperative in the indefinite games as follows:

\[
\text{AD} \ll \text{STFT} \ll T6 \ll T7 \ll T8 \ll \text{Grim} \ll \text{TFT} \ll \text{Grim2} < \text{TF2T} < \text{AC},
\]

where \( \ll, \ll \), and \( < \) represent domination in the orders of \( \varepsilon^0 (= 1) \), \( \varepsilon \), and \( \varepsilon^2 \), respectively.

### A.1.2 General implementation errors

When the probability \( \varepsilon \in \left[0, \frac{1}{2}\right] \) of implementation errors is not necessarily small, the cooperativeness of the strategies TFT, STFT, Grim, Grim2, and TF2T can be computed numerically using (1) for the finite games and by (2) for the indefinite games, whereas the cooperativeness of AC and AD equals \( g_{CC} \) and \( g_{DD} \), respectively, as above. Consider now the strategy \( T_k \) \((k = 6, 7, 8)\). In the indefinite games, its cooperativeness can be computed as

\[
v^{T_k} = (1 - \delta) \frac{1 - (\delta p)^{k-1}}{1 - \delta p} g_{CC} + \delta \left\{ (1 - p) \frac{1 - (\delta p)^{k-2}}{1 - \delta p} + (\delta p)^{k-2} \right\} g_{DD}.
\]

In the finite games, suppose that \( t < k \) and let \( v_t \) denote the sum of stage payoffs in rounds \( t, t+1, \ldots, 8 \) when \( T_k \) still specifies action \( C \) in round \( t \). We have the following recursive equations:

\[
\begin{align*}
v_{k-1} &= g_{CC} + (9 - k) g_{DD}, \\
v_{k-2} &= g_{CC} + pv_{k-1} + (1 - p)(10 - k) g_{DD}, \\
& \quad \vdots \\
v_2 &= g_{CC} + pv_3 + (1 - p) \cdot 6g_{DD}, \\
v_1 &= g_{CC} + pv_2 + (1 - p) \cdot 7g_{DD}.
\end{align*}
\]

The cooperativeness of \( T_k \) then equals \( v^{T_k} = \frac{v_8}{8} \).