Critical thinking about "critical loss" in antitrust

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I. Introduction

Since first proposed by Barry Harris and Joseph Simons, the critical loss test has been often used as one of the techniques to define antitrust markets.¹ Most recently it played a key role in

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three judicial decisions involving hospital mergers in Dubuque, Iowa; Poplar Bluff, Missouri; and Berkeley/Oakland, California.\(^2\) In all three opinions, the court rejected the market definition proposed by the federal or state antitrust authority. In the more typical setting of merger review, critical loss can play a major role in the thinking of antitrust authorities, consultants and researchers studying the economic consequences of a merger. Yet, in spite of the practical importance of this technique, to date there is only a small literature commenting on it.\(^3\)

The critical loss test is closely tied to the Horizontal Merger Guidelines. The Guidelines define an antitrust market as a product or group of products and a geographic area in which it is produced or sold such that a hypothetical monopolist would raise price by a "small but significant and nontransitory" amount. The relevant market generally is a group of products and a geographic area that is no bigger than necessary to satisfy this test. In most contexts, the hypothetical price increase is taken to be 5% and the time horizon is the foreseeable future.\(^4\)

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The critical sales loss is the decrease in sales resulting from a particular price increase that is just large enough to make that price increase unprofitable. Once computed, the critical loss is compared to the sales loss expected following the particular hypothetical price increase. If the actual sales loss is expected to be larger than the critical loss, it follows that the price increase would be unprofitable. This thought experiment connects to antitrust market definition in the following way: if the hypothetical price increase is deemed unprofitable then the candidate market is considered to be too narrow and is expanded to account for sales lost to producers located outside the candidate market.

This article, broadly consistent with the ideas and findings expressed by Langenfeld and Li in the same issue of this journal, critiques critical loss analysis on somewhat different conceptual grounds. First, we show that, under certain circumstances, commonly applied critical loss techniques are inconsistent with the frequent claim—typically made by the merging parties—that the market for the product is “exceptionally competitive.” Second, critical loss techniques, even at best, are sensitive to the degree of preexisting market power. Third, large differences between price and marginal cost ordinarily imply that few sales are expected to be lost when price is increased even though the critical sales loss

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5 Specifically, this definition applies to breakeven critical loss. Also in common use is the profit-maximizing critical loss approach derived by Werden (see Werden, supra note 3, Four Suggestions on Market Delineation). Most of our comments apply to both approaches.

6 Werden notes that the price increase postulated in breakeven critical loss analyses is often inappropriate because the hypothetical monopolist’s optimal price increase can be more or less than the particular price increase assumed. When the optimal price increase exceeds the assumed price increase, the market may be drawn too large and vice versa. This problem forms the basis for his profit-maximizing version of the critical loss analysis. (See Werden, supra note 3, Four Suggestions on Market Delineation).

7 One could also compute a critical loss for analyzing competitive effects. This is not necessarily identical to the market definition critical loss. For example, a merger may not involve all the firms in the antitrust market.
is low. Fourth, accounting data often are poor and obtaining good estimates of marginal cost from such data is especially difficult. Fifth, the relevant time horizon for calculating marginal cost to use in estimating the critical loss can be far longer than that implied by the commonly used concept of accounting average variable cost.

II. A summary of the critical loss technique

Harris and Simons illustrate critical loss theory by applying it to a hypothetical monopolist, formed by the merger of numerous firms that were initially in competitive equilibrium. The situation faced by the hypothetical monopolist immediately after the large-scale merger is depicted in figure 1, which reproduces their figure 1. The cost curves are the sum of the short-run costs of the original firms, hence they represent the short-run cost curves of the newly-created hypothetical monopoly.

Premerger profit earned by the hypothetical monopolist is written in terms of average variable cost (equation numbering follows Harris and Simons)

\[(HS \#1) \quad \text{Profit}_0 = P_0Q_0 - Q_0^*AVC_0 - \text{Fixed Costs.}\]

Next, in this thought experiment, the monopolist raises its price, exploiting its newly-created market power. This is portrayed in figure 2, also reproduced from Harris and Simons.

After the monopolist has exercised its monopoly power, profit is written as follows,

\[(HS \#2) \quad \text{Profit}_1 = P_1Q_1 - Q_1^*AVC_1 - \text{Fixed Costs.}\]

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8 See Harris & Simons, supra note

9 The firms are assumed to have identical prices and cost curves, implying either homogeneous goods or a strong form of symmetry similar to that of monopolistically competitive models. See Dennis Carlton & Jeffrey Perloff, Modern Industrial Organization, chap. 7 (2000).
The critical loss in sales is the loss that would make firms indifferent between market prices $P_0$ and $P_1$. In other words, the critical loss in sales returns the hypothetical monopolist to the premerger profit level. Algebraically,

$$P_0Q_0 - Q_0^*AVC_0 - \text{Fixed Costs} = P_1Q_1 - Q_1^*AVC_1 - \text{Fixed Costs}.$$ 

The fixed costs in the above expression, of course, cancel since postmerger fixed costs are assumed to be identical to the premerger level. Making the assumption that average variable costs (and hence marginal costs) are quite flat over the relevant
region, Harris and Simons substitute AVC_0 for AVC_1.\textsuperscript{10} This assumption, however, is far from innocent and receives a substantial critique from Langenfeld and Li.\textsuperscript{11} In any case, after some algebraic manipulation they find that the critical loss in sales, X, is expressed (as a percentage) as follows:

\begin{equation}
\text{(HS \#13)} \quad X = \left[\frac{Y}{Y + CM}\right] \times 100,
\end{equation}

where Y is defined as the percentage increase in price over the premerger level and CM is defined as the contribution margin.

\textit{See} Harris & Simons, \textit{supra} note 1, at 214, n.17.

\textit{See} Langenfeld & Li, \textit{supra} note 3.
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(i.e., the extent to which price exceeds the cost, measured as a percentage of the price). The Y and CM are defined as follows:

\[ Y = \left( \frac{P_1 - P_0}{P_0} \right) \times 100, \]
\[ CM = \left( \frac{P_0 - AVC_0}{P_0} \right) \times 100. \]

Note that the contribution margin is defined as price less average variable cost, not marginal cost. The conceptually correct contribution margin is defined over marginal cost not variable cost. In order to substitute average variable cost for marginal cost, average variable cost must be constant over the relevant range.

The immediate appeal of this critical loss equation lies in its clarity and simplicity. Following Harris and Simons’s example, suppose the relevant contribution margin in aluminum production is 50% and one is investigating a 5% price rise as a “small, but significant and nontransitory” price increase. The critical loss is computed as follows:

\[ X = \left( \frac{Y}{Y + CM} \right) \times 100, \]
\[ = \left( \frac{5}{5 + 50} \right) \times 100, \]
\[ = 9.1. \]

Thus, if a hypothetical 5% price increase is expected to result in more than 9.1% of current sales being lost by the monopolist, then the price increase is unprofitable. The lost sales hurt profits more than the higher price helps profits. In turn this implies that the candidate market is too small and the market must be expanded to include the products or geography to which many of these sales are lost.

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12 See Harris & Simons, supra note 1, at 215.

13 Note an interesting complication that we do not pursue. Depending on the shape of the demand and cost curves, a market might be rejected with a critical loss analysis for a 5% price increase, but accepted for a 10% price increase (see Werden, Demand Elasticities in Antitrust Analysis, supra note 3).
III Critical loss in theory and practice

A. Perfect competition

The parties to a merger often claim that the market is "exceptionally competitive." Although the precise meaning is uncertain, suppose we interpret this as meaning perfectly competitive pricing. If literally true, then price not only equals average cost, but it also equals marginal cost (see figure 1). Using marginal cost at the starting point, the contribution margin is zero and the critical loss is 100%. That is, when the monopolist raises price it would have to lose all of its sales in order for there not to be an increase in profits! Even allowing for nonconstant marginal costs, it is generally true that a hypothetical monopolist can always increase profits by raising price and reducing output, starting from a competitive initial equilibrium.

The lesson here is obvious and important. At a very general level, when markets are exceptionally competitive, price can be close to marginal cost. When this is true, the contribution margin is small and the critical loss is large. That is, the hypothetical monopolist can afford to lose many sales when it raises price and still have profits increase. As a result, critical loss analysis may define the antitrust market quite narrowly. Thus, the parties' arguments that the market is exceptionally competitive may be at odds with a broad market definition.

It is equally important to note that the practice of replacing marginal cost with average variable cost favors the finding of a broad market. To see this, refer to figure 1, which shows that average variable cost is below marginal cost at the starting point, Q₀. Thus, by replacing marginal cost with average variable cost the contribution margin is biased up and the critical loss down, since average variable cost is less than marginal cost at the competitive

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14 Generally, critical loss calculations are sensitive to the shape of the marginal cost curve.

15 Indeed, in the typical situation, marginal cost is above average variable cost. One notable exception can occur when there are increasing returns to scale in the relevant region.
starting point. While this substitution is sometimes made in practice because estimating marginal cost is difficult, it may substantially affect the estimate of critical loss and therefore the market definition.

B. Imperfect competition

Suppose that we reformulate the analysis for a hypothetical merger where the starting point is some degree of monopoly, but long-run equilibrium still prevails. For example, consider a Cournot equilibrium with barriers to entry. Abstracting from non-constant returns, in this situation, costs are minimized before the hypothetical monopolist raises price. But, because of the preexisting imperfect competition, price exceeds marginal costs (see figure 3).

Figure 3
The Imperfectly Competitive Situation Inherited by the Monopolist

![Diagram](image-url)
Extending Harris and Simons's example, suppose that the marginal cost of the merged aluminum firms is 80% of the price because price was already marked up over marginal cost. The contribution margin is 20% and the critical loss for a 5% price increase is given by

\[
X = \frac{Y}{Y + CM} \times 100,
\]

\[
= \frac{5}{5 + 20} \times 100,
\]

\[
= 20.
\]

The lesson to be learned here is that when price exceeds marginal cost, losing volume is more painful because profits are lost on the sales given up. In turn, the critical loss is low when the contribution margin is high.

Notice in figure 3, however, that marginal cost at the starting point is still systematically underestimated by average variable cost. This results from the fact that, even with market power, firms will adjust all inputs such that average total cost is minimized. At that production point, average total cost is equal to marginal cost. And, since average variable cost excludes some costs that are included in the total cost curve, it will necessarily be below marginal cost. The fact that average variable cost is below marginal cost comes from nothing more than cost minimization, which is a necessary component of profit maximization, and constant returns.16

CRITICAL LOSS IS SENSITIVE TO STARTING MARKET POWER As the above analysis illustrates, even when using the theoretically correct concept of marginal cost, market definition via critical loss is highly sensitive to the degree of market power at the starting point. More preexisting market power leads to smaller critical loss estimates, thus potentially broader market definitions.

To clarify the idea, consider the limiting case of a market composed of two firms that are already colluding. If price is already at

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16 Strongly nonconstant returns can, in some circumstances, reverse this outcome.
the monopoly level, then any further increase in price will result in lower profits. The critical loss is zero. So, the expected loss would necessarily be greater than the critical loss for any price increase. In this situation the critical loss analysis would suggest that the group of products over which the two firms were successfully colluding is not a relevant antitrust market, when in fact it obviously is one. This is an example of the cellophane fallacy where the analyst confuses substitution induced by monopoly prices with substitution that would preclude monopoly pricing.17

C. Critical loss analysis may lead to an overbroad market definition when marginal cost is low

Consider a situation that would likely be touted as being favorable to the merging parties. In this situation, in the absence of collusion, the contribution margin on the last sale is large and, as a result, the last sale contributes greatly to profits. Normally the merging parties would be expected to argue that the profitability of those marginal sales reduces the incentive of the hypothetical monopolist to raise price and potentially lose those sales. As a result, the parties are likely to argue that the critical loss is low and the market large. But, the calculation of a low critical loss doesn't necessarily lead to a broad market definition. One must keep in mind that the critical loss must be compared to the expected actual loss.

A high markup of price over marginal cost implies that demand is relatively inelastic at the level of the firm. This sug-

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17 The cellophane fallacy generally leads to an excessively broad market definition. Except for a very brief note, the Guidelines ignore the cellophane problem. This may lead to the right policy decision on a merger even though the market definition is wrong. The cellophane fallacy may be especially important in civil, nonmerger situations, such as price fixing. Posner argues that the fallacy is unimportant for mergers, but this seems to confuse market definition with monopolistic effect. See Richard A. Posner, Antitrust Law: An Economic Perspective 127–29 (1976). For an analysis in terms of market definition, see Gene C. Schaerr, The Cellophane Fallacy and the Justice Department's Guidelines for Horizontal Mergers, 94 Yale L.J. 670 (1985).
gests that little volume would be lost as a result of a price increase by the hypothetical monopolist. Indeed, findings that a price increase would cause large losses in sales are logically inconsistent with the contention that marginal cost is low and the contribution margin is large.

To fix ideas, consider the aluminum firms of Harris and Simons’s example. We can calculate the premerger elasticity of demand for the original firms using the following equation

\[
\text{Lerner Index} = \frac{P - MC}{P} = -1
\]

If marginal cost equals 50% of price, the equation shows that the elasticity of demand is −2.0. This means that if a firm raised price by 5% its demand would decline by only 10%.

In this thought experiment, some of the sales lost by the firm go to other firms producing related goods and some consumers simply stop purchasing. Similarly, when all firms in a candidate market raise price, some of the sales lost go to other related goods, not contained within the market as well as those consumers who simply stop purchasing. But, there is an important difference: when all firms raise price, consumers don’t have the luxury of switching to a low priced firm within the market and as a result one would expect fewer sales to be lost when all firms raise price than when a single firm raises price. The market demand curve is less elastic than the individual firm’s demand curve. Therefore, a 5% increase in price for all goods will reduce demand by less than 10%.

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18 The statement is true for demand curves without kinks. If there is a kink at a price slightly higher than the initial price, the analysis of the expected actual loss is more complex.

19 The situation becomes more complex when strong symmetry is not imposed on the product since the monopolist may want to raise the price of some products more than other products.
What would the elasticity of market demand have to be to make a 5% price increase by the hypothetical monopolist profitable? Solving for the required elasticity gives –1.91. The required difference in elasticity is small, only –0.09. It seems that could often be easily achieved by merging firms that are reasonably close competitors. If so, the candidate market definition would be accepted because the expected loss is less than the critical loss. As Langenfeld and Li illustrate, in different language, this point seems to have been missed by some courts.20

D. Estimating critical loss using accounting data

Harris and Simons suggest taking average variable cost directly from accounting reports. They say that most companies maintain data allowing the calculation of average variable cost at the current output. This is not so simple. The only data commonly (though not universally) available in accounting reports that resembles average variable cost is the concept of “cost of goods sold,” or “product costs.” In manufacturing firms, this consists of fixed and variable manufacturing costs, and excludes all other costs.21 This is the only concept related to variable costs that is allowed by generally accepted accounting practices.22 The difference between this cost and the price is called the gross margin.

There is wide recognition that the traditional approach doesn’t estimate true variable costs well, leading analysts to more advanced cost accounting techniques. These techniques, based partly on the subjective judgments of managers and analysts, and partly on

20 See Langenfeld & Li, supra note 3

21 Confusingly the cost of goods sold, often viewed as an estimate of variable costs, includes fixed manufacturing costs. The costs that are left out of cost of goods sold are called the fixed costs. Many important costs are in this category, including general and administrative and marketing labor, some buildings, machinery, electricity, etc. Some of these costs are called, loosely speaking, overhead.

statistical analyses, partition all costs, including overhead costs, according to whether or not they are variable or fixed.\textsuperscript{23} Taking a more long-run approach, activity-based accounting is also becoming more prominent.\textsuperscript{24} But, accounting reports based on these more advanced methods are not universally available. Because of the partly subjective nature of the cost studies and the often relatively crude estimates used in these calculations, their quality varies.\textsuperscript{25} However, the inaccuracy of cost accounting may be a smaller problem than the issue of time horizon for critical loss calculations for antitrust.

\textbf{E. What is the relevant time horizon for market definition?}

The relevant time period for the firm’s pricing decisions and also the relevant horizon for antitrust concerns are important issues for defining antitrust markets. In the long run, demand elasticities can be different than in the short run,\textsuperscript{26} but, more importantly, marginal costs are usually very different for different decision-making time horizons.

\textsuperscript{23} \textit{Id.} at 309–30.

\textsuperscript{24} \textit{Id.} at 231–66. For applications to health care providers see \textsc{Steven A. Finkler} \& \textsc{David M. Ward}, \textsc{Essentials of Cost Accounting for Health Care Organizations} 366–72 (2d ed. 1999); \textsc{Judith J. Baker} \& \textsc{Georgia F. Boyd}, \textit{Activity-Based Costing in the Operating Room at Valley View Hospital}, in \textsc{Issues in Cost Accounting for Healthcare Organizations} 365–70 (\textsc{Steven A. Finkler} \& \textsc{David M. Ward}, 2d ed. 1999); and \textsc{Suneel Udpa}, \textit{Activity-Based Costing for Hospitals}, in \textit{id.} at 371–84.

\textsuperscript{25} In activity-based costing, expense in a single overhead category is often assumed to be caused entirely by a single cost driver. The cost per unit of driver is then estimated by dividing that total overhead expense by the total level of the driver. In variable costing, managerial and analyst opinion is sometimes used to decide which part of an overhead expense is variable. Sometimes simple regression of the expense on a single measure of volume is used. Cost accountants argue that these crude and subjective approaches make economic sense, because more accurate information may not be worth the extra cost. \textsc{See Maher, supra note 22, at} 71–72 and \textsc{Finkler} \& \textsc{Ward, supra note 24, at} 8.

\textsuperscript{26} For example, long-run demand can be either more elastic or less elastic than short-run demand.
It is common to approximate marginal cost with average variable cost where variable costs are defined as costs that vary with output. In such instances, data are typically taken from accounting reports. This definition implicitly takes a short-run approach. This common approach may be inappropriately shortsighted for antitrust, especially for merger analysis.

Another definition that implicitly defines variable costs is as follows: factors of production are fixed when it is not profitable to vary them at short notice. This definition explicitly acknowledges that there is a time dimension to fixed costs and that few factors are likely to be fixed in the longer term. While the definitions are equivalent for identical time horizons, the differing language suggests different time horizons, thus different predictions of the relevant marginal cost.

In focusing on some measure of accounting variable costs, the implicit assumption is that the “fixed” factors cannot be varied in a time period relevant to antitrust concerns generally or mergers in particular. Thus, any monopolistic output reduction would not avoid the “fixed” costs and, therefore, would not save much on costs. This reasoning leads to a low estimate of marginal cost, a high contribution margin, a low critical loss and potentially broadly defined markets.

But this approach is often inappropriate. In general, expenditures on so-called fixed factors can be part of the marginal cost that is relevant to antitrust law. The question is: What is the time horizon that is relevant for pricing and output decisions? Consider a hypothetical merger. How does the merged entity respond to its newly-created market power? In the long run, the answer is simple. It maximizes profits, given its long-run costs. Because in the long run all inputs are variable, all costs are variable costs. Thus, for long-run analysis, the relevant marginal cost is long-run marginal cost. This may lead to low contribution margins, large critical loss numbers and relatively narrow market definitions.

In the short run, if the hypothetical merger is expected to lead only temporarily to market power (e.g., if patents expire in a year), the situation is different. Here, the relevant marginal cost
The antitrust bulletin concept is short run. It is typically too costly to vary such overhead inputs as general and administrative labor or capacity. This will lead to high contribution margins, small critical loss numbers and relatively broad market definitions.

But, market definition is based on a hypothetical merger that exists for the foreseeable future. In this situation, the hypothetical monopolist is free to optimally reduce so-called fixed costs as it reduces output (e.g., closing plants, leasing buildings, reducing administrative staffing). Thus, the merged entity can respond according to its long-run marginal cost. This long-run approach is recommended in the cost accounting literature. For example, Michael Maher suggests using short-term costs as a basis for pricing only for special, one-time-only transactions that have no long-term implications. He suggests using long-term costs for pricing a main product. Further, surveys of actual firms show a consistent pattern, both in the U.S. and abroad. Most firms use full cost, not variable cost, in their pricing decisions. Many use activity-based accounting, discussed above, which is a variant of full costing.

IV. Conclusion

The critical loss analysis widely used for market definition in U.S. antitrust cases is problematic for several reasons. When the market is initially competitive, the critical loss technique may lead to narrow market definitions. In this context, the contention that the market is competitive may be at odds with the contention that the market is broad. When the market is initially less than competitive, the technique, in a variant of the cellophane fallacy, tends to give overly broad markets.

Another problem arises when the market is initially less than competitive because of relatively inelastic demand at the level of

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27 But, in the short run, the extent of the output contraction matters. The greater the output contraction, the more likely some components of "fixed costs" can be avoided and hence marginal cost is higher.

28 See Maher, supra note 22, at 421, 424 & 427.
the firm. Here, the critical loss may be truly low, yet the demand facing a hypothetical merged entity can be so inelastic that the expected actual loss is even lower. If so, a low critical loss doesn’t signal that market definition needs to be broader.

Finally, the short-run variable cost concept commonly used to estimate marginal cost assumes a time horizon that may be too short, leading to overly broad market definitions. Within the longer time horizon that’s relevant to antitrust issues, especially mergers, many of the “fixed” costs on accounting statements may become variable.