1 Voting–In praise of democracy?

> "Many forms of Government have been tried, and will be tried in this world of sin and woe. No one pretends that democracy is perfect or all-wise. Indeed, it has been said that democracy is the worst form of government except all those other forms that have been tried from time to time"..., Winston Churchill, 1947

What a bunch of pompous talk... and how frequently it is quoted! Where is the evidence? What do we mean by democracy? Churchill weasels by saying "it has been said that" If someone else said it, why doesn't he or anybody else figure out who said it? There does seem to be a ring of truth here, but the quotation is not particularly useful in focusing on the right kinds of questions.

We have some good news and some bad news. Lets start with the bad news.

1.1 Bad News–The Condorcet Paradox

Marie Jean Antoine Nicolas de Caritat, Marquis de Condorcet was born in 1743 and died in 1794. He advocated a liberal economy, free and equal public education, constitutionalism, and equal rights for women and people of all races. He is said to embody the ideals of rationalism and the French Enlightenment. Played an active role in French politics during the revolution (Girondist) He died in prison after being a fugitive from French Revolutionary authorities.

Condorcet pointed out that "defeats by pairwise majority voting" is an intransitive relation. The canonical example has 3 voters and 3 candidates, A, B, and C. Voter 1's preference ordering of the candidates is ABC. Voter 2's is BCA, and voter 3's is CAB. In pairwise majority voting, A beats B, B beats C, but C beats A. So there is no candidate who will defeat all challengers by majority vote. If candidates are voted and eliminated in succession, as in parliamentary procedure, then the outcome depends on the order in which options are introduced. For the example just described, if you have a candidate whom you want to win, find the candidate who can beat your candidate. Make the first vote a contest between this candidate and the candidate who can beat him. Now run the winner of this contest against your chosen candidate. Your candidate will be the winner.

2 Worse news: Arrow's Impossibility

Let us assume that there is a social welfare function that maps individual ordinal preferences over alternatives into a social ranking and let us suppose it has the following properties.

Transitive

Universal domain

Pareto principle

Independence of irrelevant alternatives

The only function that satisfies these properties is one in which the social ordering is the same as that of some member of the society. (a dictatorship)

Proof: See Geanakoplos

2.1 Good News? Borda Count

Jean-Charles, Chevalier de Borda (1733 1799) was a French mathematician, physicist, political scientist, and sailor. He was a military engineer who wrote on theory of projectiles. He was elected to the French Academy of Sciences in 1764. Between 1777 and 1778, he participated in the American Revolutionary War. In 1781, he was put in charge of several vessels in the French Navy. In 1782, he was captured by the English, and was returned to France. He returned as an engineer in the French Navy, making improvements to waterwheels and pumps. He was appointed as France's Inspector of Naval Shipbuilding in 1784, introduced a massive construction programme to revitalise the French navy Another of his contributions is his construction of the standard metre, basis of the metric system.

In 1770, Borda formulated a ranked preferential voting system that is referred to as the Borda count. The French Academy of Sciences used Borda's method to elect its members for about two decades until it was quashed by Napoleon Bonaparte who insisted that his own method be used after he became president of the Acadmie in 1801.

In the Borda count, each voter submits a ranking of the candidates, with the first choice ranked 1, second choice 2, etc. The individual voter ranks are added. The winner is the candidate for whom this sum is lowest. In the event of a tie, one of the winners is chosen at random.

This method satisfies most of Arrow's axioms, but does not satisfy independence of irrelevant alternatives.

2.2 Bad news, Borda count is Not strategy-proof

This is a slight variant of a sad and partially apocryphal story told by Hugo Sonnenschein in a lecture delivered at Northwestern University. An economics department needed to pick a new department chairman. They decided to do so by means of Borda count voting. The department had 11 macroeconomists, 11 microeconomists, and a guy named Glutz. The macroeconomists all preferred a macroeconomist (call him Keynes). The microeconomists all preferred a microeconomist (call him Edgeworth). Glutz preferred himself. Everybody else thought Glutz was worse than either Keynes or Edgeworth. The three candidates were Keynes, Edgeworth, and Glutz.

They all realized that if everbody voted sincerely, the outcome would be a tie between Keynes and Edgeworth. Each macroeconomist concluded that the way to fix this would be to vote as if Edgeworth was his or her third choice. Each microeconomist concluded that the way to fix this would be to vote as if Keynes was his or her third choice. Glutz didn't think strategically. He just voted himself as his first choice, Keynes second, and Edgeworth third.

The balloting took place and the scores were Glutz 45, Keynes 46, Edgeworth 49. So to the sorrow of everyone except Glutz, the new chairman was Glutz.

2.3 Some good news–Single peaked preferences

Maybe Universal domain needn't apply. Charles Dodgson 1832-1898 was an English mathematician and logician who wrote Alice in Wonderland under the pseudonym Lewis Carroll.

This idea reappeared in the modern literature in the work of Duncan Black JPE , 1948

In 1 dimension, with single peaked preferences, there is a Condorcet winner. Define single peaked preferences— for some arrangement along line. Show that SP is implied by quasi-concave.

Show that winner is median of favorite points.

Show that Condorcet paradox example is not single-peaked.

Show that if preferences are convex and tax shares are fixed, then preferences over expenditure on a single good are quasi-concave (single-peaked). Median voter theorem.

2.4 More good news

Bowen's theorem. If distribution of willingness to pay is symmetric and public good is paid for by a head tax, then median of preferred points is Pareto optimal. Median equal mean.

If preferences are identical and Cobb-Douglas and income is not symmetrically distributed, (mean income usually exceeds mean) then head tax would not produce efficient amount but income tax would.

Suggests that attention should be paid to asymmetric distributions. Many issues are that way.

More generally, with averaged Lindahl equilibrium, bowen conclusion applies.

2.5 Some bad news–Single peaked preferences don't help when there are two or more dimensions

Show the Olympic logo diagram.

Jerry Kelly's theorem

on single-peaked voting issue by issue..

2.6 Some pretty good news

Peyton Young reports that Condorcet had a theory of voting that would apply when "enlightened voters honestly attempt to judge what decision would best serve society." Each voter is asked to supply a rank ordering or the alternatives. Suppose that we apply the ranking $>$ where $a > b$ means more voters prefer a to b than the other way around. In general $>$ will not be transitive. If there is an intransitivity, Condorcet proposes to 'throw out' (Young suggests that he realy means 'reverse') the ranking that wins by the smallest majority–and do so successively until a winner is chosen. interpretation of Condorcet's jury theory. Young believes that Condorcet was looking for the maximum likelihood "true ranking" of the candidates, given that each voter has some probability of getting the answer wrong.

Young (and probably) Condorcet figured out that if all you want to do is pick the "best" candidate, rather than supply a full ranking, the winner of the Borda count has a strong claim.

Nalebuff and Kaplan 64% majority rule result.

Uniform distribution on a convex body in n dimensions minimum stable percentage is $1 - (n/n + 1)^n$. Lim as n approaches infinity is 64