Lecture Notes on Anticommons

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These notes illustrate the problem of the anticommons for one particular example.

Sales with incomplete information

Bilateral Monopoly

We start with the case where there is no anticommons problem, since ownership of a resource is consolidated under a single original owner. Let there be one possible buyer. The seller doesn't know the buyer's willingness to pay. The seller believes that the probability that the buyer is willing to pay no more than x is $F(x)$ where F is a continuous function.

The original owner values the item at y. Suppose that the owner can post a single take-it-or-leave it price, P. The buyer will accept the offer only if his willingness to pay is at least P, so the probability of a sale is $1-F(P)$. If he gets a sale, the seller's profit from the sale is $P-y$. Therefore the seller's expected profit if he sets the price at P will be

$$(P-y)\left(1-F(P)\right).$$

Differentiating this expression with respect to P, we find that if expected profits are maximized, then

$$\frac{1-F(P)}{f(P)} = (P-y).$$

Let us look at the special case where the seller believes that the buyer's willingness to pay is uniformly distributed on the interval $[0,100]$. Then $F(P) = P/100$ for all P between 0 and 100 and $f(p) = 1/100$. The first order calculus condition for maximizing expected profit is then

$$100 - P = P - y$$

and thus

$$P = 50 + \frac{y}{2}.$$

Efficiency requires that the item be sold to the buyer whenever $x > y$. In the special case where $y = 0$, the efficient solution is always for the item to be sold. But when $y = 0$, the owner demands a price of 50 and therefore with probability 1/2, the object will not be sold. More generally, if the seller's own value for the object is y, the object will be sold only if $x > P = 50 + y/2$. Therefore the object will not be sold even though it is worth more to the buyer than to the original owner whenever

$$y < x < 50 + y/2.$$

Since the distribution of x is uniform on the interval from 0 to 100, this probability is

$$\frac{1}{100}\left(50+\frac{y}{2}-y\right)=\frac{1}{2}-\frac{y}{200}=\frac{1}{2}\left(1-\frac{y}{100}\right)>0.$$

The positive probability that the item will not be sold even if it is more valuable to the buyer than to the seller is an instance of the general inefficiency of monopoly when the monopolist doesn't know the buyer's willingness to pay.

An anticommons problem

This problem gets worse if more than one seller has "rights of exclusion." Let us consider the case where there are two landowners each of whom owns one piece of property. To make matters easy, let us suppose that the properties are worth nothing to the original owner or to anyone else unless they are combined under a single owner. There is a developer who is interested in buying the two pieces of property. The sellers don't know what the project is worth to the developer, but they each believe that his value for the project is a random variable x, that is uniformly distributed on the interval $[0,100]$. Sellers 1 and 2 simultaneously make offers p_1 and p_2. The developer decides either to accept both offers and buy the land for a total cost of p_1+p_2 or to reject both offers. He will accept the offers only if $x>p_1+p_2$. Since the distribution of x is uniform over the interval $[0,100]$, the probability that he rejects the offer is $(p_1+p_2)/100$ and the probability that he will accept the offer is

$$1-\frac{p_1+p_2}{100}.$$

Since the land is worthless to landowner 1, he will wish to maximize his expected revenue from a sale. His expected revenue from a sale is the price he demands times the probability of a sale. Therefore the expected revenue of landowner 1 is

$$p_1\left(1-\frac{p_1+p_2}{100}\right)=p_1-\frac{p_1^2+p_1p_2}{100}.$$

To maximize his expected revenue, he chooses p_1 so that the derivative of expected revenue with respect to p_1 is zero.[1] This implies that

$$1-\frac{2p_1+p_2}{100}=0.$$

Solving this equation for p_1, we have

$$p_1=50-\frac{p_2}{2}.$$

The only problem here is that in order to find out p_1, which is seller 1's offer, we need to know p_2, which is seller 2's offer. So what can we do? We perform a

[1] And he checks that the second derivative is negative.

similar exercise to solve for the the amount that seller 2 should offer given seller 1's offer.

$$p_2 = 50 - \frac{p_1}{2}.$$

When these two equations are satisfied, we have a Nash equilibrium. Solving the two equations in the two unknowns, p_1 and p_2, we find that

$$p_1 = p_2 = \frac{2}{3}50.$$

The developer will buy the land only if his value x is greater than

$$p_1 + p_2 = \frac{2}{3}100.$$

Since the buyer's value is uniformly distributed on the interval $[0, 100]$, the probability that he will buy is only $1/3$. So we get an inefficient outcome $2/3$ of the time. Recall that in the case of a single seller, the probability we get an inefficient solution "only" half the time.

Then n-sellers case

Suppose that instead of two, there are n different landowners and the buyer needs to assemble all n parcels of land to realize any value. Again, suppose that the parcels of land are of no value to the original owners. Suppose that the all of the original landowners do not know what the collection of all n parcels is worth to the buyer, but each of them thinks that the probability $F(x)$ that he is willing to pay no more than x for the entire package where $F(x) = x/(100)$. We can follow a line of reasoning very similar to that we used for the case of two landowners and we will find that the equilibrium offer for each seller i to make is

$$p_i = 100\frac{n}{n+1}.$$

In equilibrium, the sum of all the sellers' offers will be

$$P = n100\frac{n}{n+1}.$$

The probability that the buyer's willingness to pay will be less than the sum of the seller's offers is therefore

$$\frac{P}{100n} = \frac{n}{n+1}$$

and the probability that a sale takes place is only $1/(n+1)$. Therefore the larger the number of separate owners who need to be bought out, the less likely that a sale will take place. In the limit as n gets large, the probability of a sale goes to zero.

Nonzero reservation prices

We could perform a similar analysis for the case where each seller i has a positive reservation value x_i for his own parcel of land and where the x_i's which are independently distributed random variables y_i drawn from the uniform distribution on the interval $[0, 100]$. This involves a fairly complicated (but not too difficult) computation. As you might guess, the qualitative results are similar to those we found for the case where $x_i = 0$ for all i.

Anticommons with externalities and identical consumers

The Buchanan-Yoon Parking Lot

Buchanan and Yoon imagine a parking lot near the center of town. This lot congests as more people use it. Where x is the number of users, they assume that the value of using the parking lot to each user is $a - bx$ for some positive constants a and b. (The alternative to using the parking lot is to park in a large uncongested area, located a mile away.) If there are x users, each gets a value of $a - bx$ from the lot and so the total value of the parking lot to all users is $x(a - bx)$. This total value will be maximized when

$$\frac{d}{dx}x(a - bx) = 0,$$

which implies that

$$x = \frac{a}{2b}.$$

If people allowed to access the parking lot freely, they will be attracted to the parking lot so long as it is better to park there than in the uncongested area, that is, so long as $a - bx > 0$. In equilibrium we would have $a - bx = 0$ and nobody would be any better off than they would be parking a mile away.

If this parking lot were operated by a monopoly that charged everyone a price for admission to the lot, then if the monopolist could sell x tickets, he could charge $a - bx$ for each ticket since that is any individual's willingness to pay to be in the lot if there are a total of x tickets sold. So the monopolist's revenue would be $x(a - bx)$. This would be maximized when

$$\frac{d}{dx}x(a - bx) = 0,$$

which implies that

$$x = \frac{a}{2b}.$$

As we calculated above, this is also the solution that maximizes total benefit from the parking lot.

When $x = \frac{a}{2b}$, we have $p = a - bx = a/2$. Therefore the monopolist would charge a price of $a/2$ per ticket and sell $x = \frac{a}{2b}$ tickets. Since consumers are assumed to be identical, the monopolist captures the entire consumers' surplus of using the parking lot. The consumers who park in the parking lot are no better or worse off than those who park in the uncongested lot and also no better or worse off than they would be if there were free access to the lot. But with the tickets, the monopolist has a profit. (The villagers might share in this profit by selling the rights to charge for entry.)

Suppose that the local government allowed two different firms to sell tickets to the parking lot, and a person would be allowed to park if he a ticket from either seller. Let us suppose that each seller chooses the most profitable number of tickets to sell given the number sold by the other guy. If seller 1 sells x_1 tickets and seller 2 sells x_2 tickets, the total number of users of the parking lot will be $x_1 + x_2$ and the willingness to pay of any user will be $p = a - b(x_1 + x_2)$. Thus the price that either seller will get for a ticket is $a - b(x_1 + x_2)$. Revenue of seller 1 will be

$$x_1 \left(a - b(x_1 + x_2)\right).$$

To maximize his revenue, seller 1 sets the derivative of revenue with respect to x_1 equal to zero. This implies that

$$x_1 = \frac{a}{2b} - \frac{x_2}{2}.$$

Similarly

$$x_2 = \frac{a}{2b} - \frac{x_1}{2}.$$

Solving these two equations in the unknowns x_1 and x_2, we have

$$x_1 = x_2 = \frac{a}{3b}.$$

Then the total number of users of the parking lot will be

$$x_1 + x_2 = \frac{2a}{3b}$$

and the price of using the lot will be

$$a - b(x_1 + x_2) = \frac{a}{3}.$$

This is the standard Cournot duopoly solution. We note that the price is lower and the quantity sold is higher than the monopoly outcome. In the absence of externalities, this would mean that the Cournot duopoly is more efficient than monopoly. But in our model, where there are negative externalities and everybody has identical preferences, the monopoly output is efficient and the Cournot duopoly outcome has too many users.

Now suppose that two firms are allowed to sell tickets, but in order to use the lot, a commuter needs two tickets, one from each seller. Let p_1 and p_2 be the prices charged by firms 1 and 2 respectively. Then the cost to anyone of using the parking lot is $p_1 + p_2$. If x people use the parking lot, the value of using the parking lot is $a - bx$ for any user. Therefore when it costs $p_1 + p_2$ to use the lot, the number of people willing to use the lot is x where $p_1 + p_2 = a - bx$. We can solve this equation for x and we have

$$x = \frac{a}{b} - \frac{p_1 + p_2}{b}.$$

If firm 1 believes that firm 2 will charge a price of p_2, then firm 1 will choose p_1 to maximize his revenue, which is

$$p_1 x = p_1 \left(\frac{a}{b} - \frac{p_1 + p_2}{b} \right) = \frac{1}{b} \left(a p_1 - p_1^2 - p_1 p_2 \right).$$

Setting 0 equal to the derivative of revenue with respect to p_1, we have

$$0 = \frac{1}{b} \left(a - p_2 - 2p_1 \right).$$

This implies that

$$p_1 = \frac{a - p_2}{2}.$$

We have now solved for the price firm 1 will charge given the price charged by firm 2. We next solve for the price that firm 2 will charge, given the price charged by firm 1. A similar line of reasoning leads us to the equation

$$p_2 = \frac{a - p_1}{2}.$$

In equilibrium each firm has chosen its best price, given the price chosen by the other. Thus we have two equations in the two unknowns p_1 and p_2 and when we solve these two equations, we find that

$$p_1 = p_2 = \frac{a}{3}.$$

Then the total cost of parking will be $p_1 + p_2 = \frac{2}{3}a$ and the number of people who park will be

$$x = \frac{a}{b} - \frac{p_1 + p_2}{b} = \frac{1}{3}\frac{a}{b}.$$

Recall that with a monopoly, we got an efficient solution in which the price of parking was $p = \frac{1}{2}a$ and the number of people who parked

$$\frac{1}{2}\frac{a}{b}.$$

So we see that in the case where you need to buy permission from two sellers, the total cost of parking is higher and the number of people who park is lower than the efficient amount.