C H A P T E R 27

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VOTING AND EFFICIENT PUBLIC GOOD MECHANISMS

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Two neighbors are considering buying a street light. It costs $100 to install. Alice values the light at $70. Bart values it at $60. If they rely solely on the marketplace to make this decision and do not act together, neither will buy the lamp. However, it is clear that both would be better off if they acted together to make the purchase and split the cost. This street light is a public good—a good that cannot easily be denied to others once the allocation and payment for the good has been provided. Providing clear air and water or providing national security are examples of public goods. There are many others.

In this chapter, I am interested in processes for the allocation and financing of public goods. I begin by looking at markets. Market processes are widely used for the allocation of many types of goods. There are several theoretical rationales for this ubiquity. Stated concisely, they say 'Markets Work.' But, as the above example shows, markets won't necessarily 'work' when asked to allocate public goods. Instead, voting processes are widely used for that task. There is little theoretical rationale for their ubiquity. In an effort to fill the gap, I am going to look for conditions under which we can say 'Voting Works.'

1 The Story Behind 'Markets Work'[1]

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The rationale for the phrase 'Markets Work' is contained in one of the most elegant results in economics, and one of the easiest to prove—the First Fundamental Theorem of Welfare Economics. It reads: if there are enough markets and if consumers and producers take prices as given when they optimize, then Walrasian allocations (where prices for all goods are announced simultaneously and supply equals demand in all markets) are Pareto optimal.[2] An important, but often unremarked, fact is that the Walrasian allocations are also individually rational.[3] Running counter to these results is the conventional wisdom that 'in the presence of public goods, markets fail.' Such wisdom is not a falsification of the First Welfare Theorem. As we were taught by Arrow (1969), even if there are public goods and other externalities, when enough markets are set up and when agents take prices as given, market allocations will be Pareto optimal and individually rational. Nevertheless there is a basis for the conventional wisdom. It lies in the understanding, described by Samuelson (1954), that once all of the markets needed to generate a First Welfare Theorem have been created, price-taking behavior is no longer compatible with the incentives of the agents—it is not *incentive compatible*. The assumption of price-taking behavior is just not believable in this context. Free riders will inevitably arise because each consumer is a monopsony buyer of their part of the public good. So, while the First Welfare Theorem remains true with public goods, it lacks interest because it is simply not applicable to the 'real world.'

In this chapter I take incentive compatible behavior seriously. In order to say that 'a particular allocation mechanism works' I require the mechanism to be both efficient, in the sense that it select allocations that are Pareto optimal, and incentive compatible, in the sense that individuals will be willing to follow the prescribed behavior. Unfortunately finding mechanisms that are both efficient and incentive compatible is not an easy task. As Hurwicz (1972) taught us, even price-taking behavior is not incentive compatible in small private goods environments. He showed that any efficient, individually rational mechanism will not be incentive compatible. Myerson and Satterthwaite (1983) established a similar, but dual, result in a simple private good economy. They showed that any incentive compatible, individually rational mechanism will not be efficient. Hurwicz and Walker (1990) went further, dispensed with the requirement of individual rationality, and showed that there cannot be any mechanism which is simultaneously efficient and incentive compatible.

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[1] In this section I use a number of terms from the economics and mechanism design literatures without definition. I apologize to those who are not familiar with the literature but I think the basic argument of this section is understandable even if certain words are not. All terms will eventually be defined later in the chapter and examples provided.

[2] Details, required assumptions, and proofs can be found in, e.g., Mas-Colell, Whinston, and Green 1995.

[3] Individual rationality requires that all participants be left at least as well off at the chosen allocation as they would have been able to accomplish on their own with just their initial endowments. Markets do this because property rights to the privately owned initial endowments are respected.

Does this mean one should conclude that markets don't work? Economists say no by taking refuge in numbers. They note that in large private goods economies, either price-taking is approximately incentive compatible and yields efficient allocations[4] or incentive compatible behavior is approximately price-taking and yields efficient allocations.[5] I will also take this approach and say that 'a mechanism works' if it is approximately efficient and approximately incentive compatible in large economies. With this interpretation, 'Markets do Work' in private goods economies.

But large numbers don't help us with public goods. 'Markets don't Work' in public goods economies, even if they are large. For example, Roberts (1976) shows us that in large public goods economies any mechanism that is individually rational, as is certainly true of markets,[6] cannot be even approximately incentive compatible.[7] But if markets can't work, maybe something else will.

One is naturally led to ask: if we are prepared to dispense with a requirement of individual rationality, are there any mechanisms, other than markets, which 'work' in public goods economies? In the following sections I provide some answers to this question. In section 2, I look at two well-known processes for the allocation of public goods: majority rule and demand revealing processes. In section 3, I ask whether there is anything significantly better and find that, in certain large environments, the answer is no. In section 4, I look at voting in multidimensional issue spaces and explore whether there is anything better there. In section 5, I provide some aggressive conclusions. In the process I establish several conditions under which one can say 'Voting Works.'

2 IMPERFECT PUBLIC GOOD MECHANISMS

In this section I examine a number of processes that are either used or have been proposed for use in the allocation of public goods. Ultimately I end up focusing on two particularly interesting incentive compatible processes: majority rule and demand revelation. Neither of them produces efficient allocations. Neither of them guarantees individually rational allocations. They are imperfect but, as we will see later, they are about the best processes one can find.

But before I get to these processes I need to develop and illustrate a number of ideas and terms including the simple public good environment, mechanisms, consumer

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[4] See, for example, Roberts and Postlewaite 1976. Various core-limit theorems also support this view. See, for example, Bewley 1973.

[5] McAfee 1992 provides an example of an incentive compatible mechanism for a simple private good economy which is approximately efficient in large economies.

[6] The canonical version of price-taking equilibrium in markets with public goods is Lindahl equilibrium. This equilibrium does satisfy individual rationality.

[7] There is another literature which also arrives at similar results when considering the core of a public goods economy. Muench 1972 provides a demonstration that the core with public goods need not shrink as numbers grow. See also Conley 1994. Here, also, a requirement of individual rationality plays a crucial role.

behavior, mechanism performance, allocative efficiency, and incentive compatibility. Readers familiar with these concepts can skip the next subsection without missing anything necessary for the rest of the chapter.

2.1 Some Important Concepts

I start by introducing a simple *public good environment* we can use as a basis for most of this chapter. There is a public good of fixed size, it can either be produced or not, that costs K. There are n consumers, each with a utility function, $v^i y - t^i$. The parameter $v^i > 0$ is the voter's *type*. $y = 1$ if the public good is produced. $y = 0$ otherwise.[8] The amount t^i is the tax paid by the consumer. We call the vector $(y, t^1, \ldots, t^n)$ an *allocation*. An allocation is *feasible* if the sum of the taxes collected from all the consumers is at least equal to the cost of the public good. That is, $\sum_{i=1}^{n} t^i \geq K y$.

> **Example 1:** *The opening example of the street lamp fits into this framework. The good costs 100. There are two consumers, A and B. A's utility function is $70y - t^A$ and B's is $60y - t^b$.*

> **Example 2:** *Here is an example we will return to later. $K = 10$. There are three consumers numbered 1, 2, and 3. 1's utility function is $u^1 = 2y - t^1$, 2's is $u^2 = 4y - t^2$, and 3's is $u^3 = 6y - t^3$. One possible feasible allocation is $y = 1$, $t^1 = 5$, $t^2 = 4$, $t^3 = 6$ since $(5 + 4 + 6) = 15 > 10$.*

In this world, a process or *mechanism* will take information from the consumers in the form of votes, demands, value statements, or other language and pick a level of public good and a vector of taxes, one for each consumer. Each voter i provides a *message* i denoted by m^i. The mechanism then translates the set of messages into a decision concerning how much is spent on the public good and how much each voter must pay in taxes $y(m^1, \ldots, m^n)$, $t^1(m^1, \ldots, m^n)$, $\ldots$, $t^n(m^1, \ldots, m^n)$.

> **Example 3:** *A very simple, well-studied mechanism is called the voluntary contribution mechanism (VCM for short). The consumer is asked to report how much she is willing to contribute towards the public good.[9] Let that amount be m^i. VCM then produces an allocation. The good is produced if and only if the contributions add up to at least the cost of the good. That is, $y = 1$ if and only if $m^1 + m^2 + \ldots + m^n \geq K$. Each consumer pays their contribution if the good is produced. That is, $t^i = m^i y$.*

> **Example 4:** *Another simple mechanism is the cost–benefit study with proportional taxation (CBS). Consumers are asked to report how much they value the public good (their true value is v^i). If the sum of the reported values, assumed to be a measure of the social benefits of the good is bigger than the cost, K, then the public good is*

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[8] In this section and much of the rest of the chapter I could deal with more general production and utility functions than these, but this environment is sufficient for illustrating the important concepts.

[9] They cannot request compensation. Contributions are not allowed to be negative. That is, $m^i \geq 0$.

produced. Taxes are then levied proportionately to the reported benefits. So y = 1 if and only if $\sum_{i=1}^{n} m^i \geq K$. And, $t^i = \left(\frac{m^i}{\sum_{i=1}^{n} m^i}\right) y$.

I am interested in mechanisms that 'work.' One standard definition of 'work' requires a mechanism to select *Pareto optimal allocations* for all values of the parameters of the environment. The outcome from a mechanism is Pareto optimal in a particular environment if there is no way to increase the utility (welfare) of one participant without reducing the welfare of another participant. It is easy to see that, in the fixed size public good environment, an allocation $(y, t^1, \ldots, t^n)$ is Pareto optimal in the environment given by $(K, v^1, \ldots, v^n)$ if and only if (a) $y = 1$ if $\sum_{i=1}^{n} v^i \geq K$ and $y = 0$ otherwise, and (b) $\sum_{i=1}^{n} t^i = y$. We call (a) output efficiency and (b) resource efficiency. [10]

Example 5: *For the three-consumer Example 2, since $(2 + 4 + 6) > 10$, efficiency requires that $y = 1$ and $\sum t^i = 10$.*

The requirement that Pareto optimal allocations be selected is a minimal, but sensible, request. If there is a way to make everyone better off, it would seem natural to ask that the mechanism be modified to do so. But, to predict whether a mechanism will select Pareto optimal allocations, we need to factor in the *behavior* of the consumers—how they select the information they provide to the mechanism, m^i, as a function of what they know, v^i. [11] If consumers choose messages according to $m^i = b^i(v^i)$, then the outcome of a mechanism is $[y(b(v)), t^1(b(v)), \ldots, t^n(b(v))]$. Let us go back to our examples to see how this works.

Example 6: *Suppose, in the VCM, each consumer is asked to report their true value for the public good. If the consumers follow our request, then $y = 1$ if and only if $\sum_{i=1}^{n} v^i = 1$. Also, $t^i = v^i$ for each i. Notice that the VCM with this behavior is output efficient but not resource efficient in environments for which $\sum_{i=1}^{n} v^i > K$.*

Thus if we don't ask for and get the appropriate behavior, a mechanism may not select Pareto optimal allocations. But in many cases we can rescue the performance by asking for a different behavior.

Example 7: *Continuing with the VCM, suppose each consumer is asked to submit a message m^i so that $v^i \geq m^i$ and $\sum_{i=1}^{n} m^i = K$ if that is possible and to submit $m^i = 0$ otherwise. [12] Again $y = 1$ iff $\sum_{i=1}^{n} v^i \geq K$ but now $\sum_{i=1}^{n} t^i = Ky$ so that the VCM with this behavior does select Pareto optimal allocations.*

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[10] (a) represents efficiency in the public good and (b) represents efficiency in the private good.

[11] For example, in the First Welfare Theorem, we not only need to know the market process (prices and allocations are set to equate supply and demand), we also need to know how consumers behave (they use demand functions derived from utility maximization taking prices as given).

[12] This is a version of VCM with rebate. Clearly this would require some iteration to accomplish. There is a way to do this in one iteration though. Modify the VCM mechanism so $y = 1$ iff $\sum_{i=1}^{n} m^i \geq K$ and $t^i = a^i$ where $m^i \geq a^i$ and $\sum_{i=1}^{n} a^i = Ky$. Then reporting $m^i = v^i$ will produce allocative efficient allocations.

There are many combinations of mechanisms and behavior that select Pareto optimal allocations. For example lets go back to the cost–benefit study.

Example 8: *In the CBS, each consumer is asked to report their true value. In this case, the outcome will be that $y = 1$ iff $\sum_{i=1}^{n} v^i \geq K$. and $t^i = \left(\frac{v^i}{\sum_{i=1}^{n} v^i} \right) y$. Thus the CBS under truthful reporting chooses Pareto optimal allocations.*

So far it seems as though finding mechanisms to efficiently allocate public goods and their costs is pretty easy. But all of this is not of much help unless the consumers actually follow the rules we suggest. So, I want to consider mechanisms in which self-interested individuals will want to follow the prescribed behavior no matter what the others are doing. That is, I want the prescribed behavior to be a dominant strategy for the consumers. If a mechanism and behavior have this property then I say that behavior and mechanism are *incentive compatible.* We illustrate this important concept by first considering some examples of mechanisms and behaviors which are not incentive compatible.

Example 9: *A simple example of a mechanism and behavior that are not incentive compatible is the CBS in Example 4 with truthful reporting. Suppose we are in the three-person Example 2. If all consumers report the truth, $m^i = v^i$, then consumer 2, say, gets $u^2 = v^2 - \left(\frac{v^2}{v^1 + v^2 + v^3} \right) 10 = 4 - \frac{40}{12} = \frac{2}{3}$, since $v^1 + v^2 + v^3 \geq 10$. But suppose 2 were to report $m^i = 2$ instead of 4. Since $m^1 + m^2 + m^3 = 2 + 2 + 6 \geq 10$, she would now get $u^2 = 4 - \frac{2}{10} 10 = 2 > \frac{2}{3}$. She is much better off than reporting truthfully. So following truthful reporting behavior is not incentive compatible for the CBS - there are times when a consumer is better off deviating from the requested behavior.*

Example 10: *The VCM mechanism is not incentive compatible for either of the reporting schemes introduced in Example 6 and in Example 7. Using Example 2, one can see why. For the behavior where consumers are asked to report truthfully, $m^i = v^i$. If consumer 2 reports the truth and the good is produced, she will end up with utility $u^2 = 4 - 4 = 0$. But if consumer 2 acts in a self-interested manner[13] then it is easy to see that she can improve her own utility by reporting a smaller number, no matter what the other consumers report. In fact, she is always best off reporting $m^i = 0$. If all three consumers do this, the combined effect of the VCM rules and the equilibrium of self-interested behavior is that $m^1 = m^2 = m^3 = 0$, and $y = t^1 = t^2 = t^3 = 0$ even if $v^1 + v^2 + v^3 > 10$. This is the classic example of free riding behavior and its corrosive effect on Pareto optimality.*

Example 11: *For a slightly more subtle example of mechanism and behavior that produce Pareto optimal allocations but are not incentive compatible, let us return to our very first story with Alice and Bart. Here we can see the difficulty that public goods create for the incentive compatibility of price-taking behavior. In order to have enough*

[13] Whether consumers will actually behave this way is an empirical question. Since this is simply an example I am not going to worry about this here. But the interested reader can see Ledyard 1995 for a survey of what was known at that time about actual behavior in the VCM.

markets to attain allocative efficiency, we need to charge Alice a different price from Bart and then pay the supplier of the lamp an amount equal to the sum of what they pay.[14] If Alice's price is $60 and Bart's is $40 and if each acts as if these prices are fixed, then each will demand the street lamp and, since the sum of the prices covers the cost, it will be supplied.[15] But there is a problem. Bart might guess that Alice has a value of around $70 and then act as if he only had a value of $30 for the street lamp when he submits his demand. Then the only equilibrium prices would be $70 for Alice and $20 for Bart. If Alice follows the same strategy as Bart and if she guesses that his value is $50 then she will act as if she only has a value of $50. But in doing so, the only equilibrium is a price of $50 for Alice, $30 for Bart, and no purchase of the lamp. Because price-taking was not compatible with their individual incentives, the market fails to produce Pareto optimal allocations.

At this point one might well ask whether there are any mechanisms and behavior that are incentive compatible. It turns out there are many.

Example 12: *A simple example of a mechanism that is incentive compatible under truthful reporting is the random dictator mechanism with constant per capita taxation (DPCTM). The mechanism asks everyone to report the level of public good they most want. It then randomly picks one consumer, say i, produces the requested amount m^i, and taxes everyone y/n. Given these messages, if i is chosen, the mechanism provides the amount of the item to person i that i says she wants. If i is not chosen, her message is ignored. Given this utility function and the rules of the mechanism, i will always want to tell the truth. So DPCTM is incentive compatible. In our fixed size public good environment, each i would like the public good to be produced, $y = 1$, if and only if $v^i \geq K/n$. They so report and they have no incentive to deviate from this.*

Example 13: *Another simple example of incentive compatible behavior in a mechanism can be crafted from the VCM mechanism. If we request all individuals to report $m^i = 0$ no matter what their value, then this is incentive compatible. It, of course, does not select Pareto optimal allocations.*

There are more interesting incentive compatible mechanisms. Two of these are voting under majority rule and the class of demand revealing mechanisms. I turn to those now.

2.2 Majority Rule

Consider a possibly complex political process in which it is agreed a priori that everyone will pay their per capita share of the cost of the public good. Suppose the equilibrium of this process is a public good level such that no other level is preferred

[14] I am describing the Lindahl equilibrium price system here. More information on this can be found in Mas-Colell, Whinston, and Green 1995.
[15] Of course, there are many possible prices, including $50 each, that would be Lindahl equilibrium prices.

by a majority. In the simple public goods environment such an equilibrium is the public good level desired by the median voter. I think of the reduced form of this voting process as majority rule with per capita taxes (MR).

We can think of the MR mechanism as follows. Each i is asked to report their ideal point. The mechanism then selects the median m^i and charges everyone $(\frac{1}{n})Ky$.

> **Example 14:** *For the three-person environment, with per capita taxes, each consumer gets utility $u^i = v^i y - (\frac{1}{3})Ky = (v^i - \frac{1}{3}K)y$. It is easy to compute that $u^1 = \frac{-4}{3}y$, $u^2 = \frac{2}{3}y$, and $u^3 = \frac{8}{3}y$. So 1's preferred level of the public good, his ideal point, is 0, while 2 and 3 both prefer 1. Therefore, 1 is the outcome of the MR process.*

The performance of the MR mechanism is easy to determine and should be generally familiar to most readers of this paper.

Proposition 1: *The MR mechanism is not individually rational.*

From the example we can see that, in equilibrium, $u^1 = -\frac{4}{3} < 0$. So 1 is clearly worse off under this mechanism than at his initial position. He will have to be 'coerced' to pay his taxes.

Proposition 2: *The MR mechanism is incentive compatible.*

Under majority rule, if the utility functions satisfy single-peakedness, which they do for our fixed size public good environment, it is clear that no one will want to lie about their preferences. Suppose that i is the median voter. Since i is getting their most preferred allocation, i will not want to move from the median. At the same time, none of the others benefit by lying. If i's ideal point is to the right of the median voter, saying that she is further to the right than she actually is will not move the outcome. Saying she is further to the left than she actually is either does not change the outcome or changes it to a position that is worse for i. So there is no incentive to lie.

Proposition 3: *The MR mechanism does not produce Pareto optimal allocations.*

MR is resource efficient but not output efficient. The MR rule reacts to the median ideal point but output efficiency requires reaction to the mean ideal point. In our fixed size world the efficient output decision is $y = 1$ if and only if $\sum_{i=1}^{n} v^i \geq K$ or iff $(\sum_{i=1}^{n} v^i)/n \geq K/n$. But in MR, $y = 1$ if and only if the median value of v^i, call it v^m, is greater than or equal to K/n. There are only a few environments, vectors of (K, v), for which the median value, q^m, is equal to the mean. Only when the equality holds will the majority rule mechanism be efficient.[16]

One of the things that prevents voting from producing Pareto optimal allocations in this context is the restriction to per capita taxes. But, if we relax that constraint and let taxes also be part of the policy space over which votes are taken, there are in general no majority rule equilibria.

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[16] To my knowledge this was first established in Bowen 1943. I thank Marcus Berliant for pointing this out to me.

Example 15: *Suppose the outcome of a majority rule process is that the public good be produced, y = 1, and taxes are assessed as follows. Consumer 1 pays $t^1 = 10$, consumer 2 pays $t^2 = 5$, and consumer 3 pays $t^3 = 15$. Assume the total collected just covers the cost of the public good, 30. Now consider a new proposal where $y = 1$, $t^1 = 5$, $t^2 = 0$, $t^3 = 25$. Both 1 and 2 are better off under the new proposal and will now vote for it over the previous outcome. It is easy to see that any vector of taxes can be beaten by a majority this way. So there will be new equilibria.*

Either with or without per capita taxes, voting appears to fail. However, before we give up on voting, we should at least find out whether we can do any better.

2.3 Demand-Revealing Mechanisms

In a well-known collection of papers,[17] we were introduced to what are now called the Vickrey–Clarke–Groves (VCG) mechanisms. In these, each person is asked to report their demand function for (or utility function for) the public good. The good is then provided at the efficient output level for the reported functions. Each individual is assessed a tax equal to the per capita cost of the public good plus the impact they have on the rest of the group. It is this tax rule that makes the mechanism incentive compatible.

In the fixed-size public goods environment, the VCG mechanism reduces to the Pivot mechanism.[18] The rules of the Pivot mechanism are as follows. Everyone is asked to report their true value $m^i = v^i$. Then if $\sum_{i=1}^{n} m^i \geq K$

$$y = 1$$

$$t^i = \frac{K}{n} \text{ if } \sum_{j=1, j\neq i}^{n} \left(m^i - \frac{K}{n}\right) \geq 0$$

$$t^i = \frac{K}{n} - \sum_{j=1, j\neq i}^{n} \left(m^j - \frac{K}{n}\right) \text{ if } \sum_{j=1, j\neq i}^{n} \left(m^j - \frac{K}{n}\right) < 0$$

and if $\sum_{i=1}^{n} m^i < K$

$$y = 0$$

$$t^i = 0 \text{ if } \sum_{j=1, j\neq i}^{n} \left(m^i - \frac{K}{n}\right) < 0$$

$$t^i = \sum_{j=1, j\neq i}^{n} \left(m^j - \frac{K}{n}\right) \text{ if } \sum_{j=1, j\neq i}^{n} \left(m^j - \frac{K}{n}\right) \geq 0$$

[17] Vickrey 1961, Clarke 1971, and Groves 1973 are all credited with independently discovering a special class of dominant strategy mechanisms once called the demand-revealing mechanisms. They are now widely known as VCG mechanisms.

[18] See Green and Laffont 1979 for a full discussion of the Pivot mechanism in this very simple environment.

Basically this says that i pays (or receives) an extra tax (subsidy) only if they are pivotal; that is, only if their presence causes the public good decision to be changed from what the others would do. If they are pivotal they must compensate the others for that change. This has the effect of converting their utility function to $(v^i + \sum_{j=1, j \neq i}^n m^j - K)y$ which, if everyone tells the truth, is $(\sum_{i=1}^n v^i - K)y$. It is thus a dominant strategy to report the true v^i.

> **Example 16:** *Consider a three-consumer example,[19] with $K = 18$, $v^1 = 2$, $v^2 = 8$, $v^3 = 10$. the VCG mechanism chooses $y = 1$ because $2 + 8 + 10 > 18$. Taxes are figured as follows. For person 1 for whom $v^1 = 2$, $\sum_{j=1, j \neq 1}^n \left(m^j - \frac{K}{n}\right) = v^2 + v^3 - 2(18/3) = 12 > 0$ so 1 is not pivotal and therefore $t^1 = 6 = (18/3)$. For person 2 for whom $v^2 = 4$, $\sum_{j=1, j \neq 2}^n \left(m^j - \frac{K}{n}\right) = (2 - 6) + (10 - 6) = 0$ so 2 is also, just, not pivotal. So $t^2 = 6$. For person 3 for whom $v^3 = 6$, $\sum_{j=1, j \neq 3}^n \left(m^j - \frac{K}{n}\right) = (2 - 6) + (4 - 6) = -4 < 0$ so 3 is pivotal. So $t^3 = 6 + 4$. One thing to note is that the total tax collected is $18 + 4 > K$.*

It is easy to prove that this mechanism is incentive compatible.

> **Lemma 1:** *VCG is dominant strategy incentive compatible with truthful behavior, $m(v) = v$.*

For completeness, we add

> **Lemma 2:** *The allocations of the VCG mechanism are generally not individually rational.*

Because it is incentive compatible, the output rule will generate allocations that are output efficient. It is sometimes erroneously claimed that 'the VCG mechanism is efficient.' That is certainly not true if by efficient we mean, as we should, that the outcomes are Pareto optimal. Because the VCG mechanism, as described above, collects more in taxes than necessary and must throw the surplus away so as to preserve incentive compatibility, the mechanism is not resource efficient and therefore is not Pareto optimal.

> **Lemma 3:** *The allocations of the VCG mechanism are output efficient but are generally not resource efficient.*
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> *Proof: The allocations are output efficient because $m(v) = v$ and, so, $y(m) = \arg\max_{y \geq 0} \sum_i [u^i(y, m^i) - ky]$. To see why they are generally not resource efficient, note that an individual's taxes are $t^i(v) = ky(v) + S^i(v_{-i})$. So $(\sum t^i) - C(y) = \sum S^i$ where*
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> $$\sum S^i(v) = \sum \left\{ \left[\max_{y \geq 0} \sum_{j \neq i}(u(y, m^j) - ky) \right] - \left[\sum_{j \neq i}(u(y(m), m^j) - ky(m)) \right] \right\}$$
>
> *It is obvious that $S^i \geq 0$ for all v. It is easy to construct examples to confirm that, generically, $S^i(v) > 0$ for every i.*

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[19] I choose different values of K and v^i here because if the values are $K = 10$ and $v^i = 2, 4, 6$ then no one is pivotal and all pay only 10/3. This is uncommon and so I change the example slightly.

So we have two interesting but imperfect processes: majority rule and VCG. Both are incentive compatible (under dominant strategies) and neither produce fully efficient (Pareto optimal) allocations.[20] This should not be a surprise. We know that there are no processes which are simultaneously dominant strategy incentive compatible and efficient over a reasonable range of possible environments even if we restrict those environments to ones with quasi-linear and quasi-concave utility functions.[21]

This leaves open at least two questions. Which of the two dominant strategy processes, VCG or MR, is the better? Is there a 'best' dominant strategy mechanism? We turn to the first of these next.

2.4 Which is Better: MR or VCG?[22]

We already know that neither the VCG mechanism nor MR is efficient. Is it possible, however, that one of them is better than the other? One result that is easy to prove suggests that MR is better than VCG.

Lemma 4: *Majority rule majority dominates the VCG process.*[23]

Proof: I will only consider the case for which $\sum m^i > K$, the case in which VCG produces the public good. A symmetric argument works for the opposite situation. (Case 1) If the median value, v^m, is larger than k, then MR would also produce the public good and everyone would pay k. Since, in the Pivot mechanism, everyone pays k plus possibly a (positive) pivot tax, everyone will be at least as well off with majority rule. (Case 2) If $v^m < k$, then MR will not produce the good and no one pays any taxes. Those for whom $v^i > k$ are worse off under MR. Those for whom $v^i < k$ are better off under MR. Under the pivot rules they don't pay a pivot tax but they are forced to consume the public good. Since $v^m < k$, there are a majority of i such that $v^i < k$. QED

But if I look at large environments I can provide a result that suggests that VCG is better than MR.

[20] For symmetry, we should point out that there are processes which are efficient and not incentive compatible. In the simple environments, one such is the direct revelation mechanism which picks the output efficient allocation of the public good, y^*, and then charges per capita taxes, $t^i = ky^*$.

[21] This was established in a sequence of papers. Green and Laffont 1977 and Walker 1978 show that the only mechanisms for public goods which are simultaneously dominant strategy incentive compatible and output efficient are VCG mechanisms. Walker (1980) and Hurwicz and Walker (1990) show that, generically in environments, VCG mechanisms are not resource efficient. Hurwicz and Walker (1990) also show this impossibiltiy is true whether we are in a public goods economy or in a private goods economy.

[22] Much of this section is based on my joint work with Ted Groves. The paper that is most closely related is Groves and Ledyard 1977.

[23] In our 1977 paper raising questions about the viability of the VCG mechanism (then called the demand-revealing mechanism), Ted Groves and I showed this result for an environment with a variable size public good where $u^i = \theta^i \ln y$ for each consumer. I suspect it is true for any quasi-linear, quasi-concave utility functions for a single public good, but that remains to be proven. This result is of course not true (or, at the very least, is generically vacuous) for multidimensional public goods since MR equilibrium rarely exist. See for example McKelvey and Schofield 1987. We address this issue later in this chapter.

Lemma 5: *In large economies, without too many consumers at extreme values[24] of v^i, VCG is almost efficient. In large asymmetric economies[25] MR is generally not efficient.*

Proof: (for VCG) I will show that as $N \to \infty$, the total pivot taxes paid goes to zero. For an economy of size N let $a(N) = (1/N)(\sum v^i) - k$. For each i, let $c^i(N) = v^i - k$. Then, given a, there are two situations in which pivot taxes are paid by i. (1) $a > 0$, $c^i > Na$ where i pays $(c^i - Na)/(N-1)$ and (2) $a < 0$, $c^i < Na$ where i pays $(Na - c^i)/(N-1)$. Let's look at the case when $a > 0$. Then the total pivot taxes paid will be $\sum_{i \ni c^i > Na}(c^i - Na)/(N-1) = \frac{\#K(Na)}{N-1} \frac{\sum_{K(Na)}(c^i - Na)}{\#K(Na)}$ where $K(Na)$ is the number of i such that $c^i > Na$. Suppose that, as $N \to \infty$, the distribution of the c^i approaches $F(\cdot)$. So $\#K(Na)/(N-1) \to 1 - F(Na)$. Also $\frac{\sum_{K(Na)}(c^i - Na)}{\#K(Na)} \to$ expected value of $c^i - Na$ conditional on $c^i > Na$. that is it approaches $\frac{1}{1-F(Na)} \int_{Na}(c - Na)dF$. So the total pivot taxes approach $\int_{Na}(c - Na)dF$. If $\int_x cf(c)dc \to 0$ as $x \to \infty$, then $\int_{Na}(c - Na)dF \to 0$ as $N \to \infty$.

(For MR) If the median value of v^i, $v^m \neq \bar{v}$, the mean value of v^i, then MR is not efficient.

At this point the situation is a bit confusing. MR is majority preferred to VCG but, in large economies, VCG produces a higher level of aggregate welfare, $\sum v^i y(v) - t^i(v)$. So which is better?

2.5 Which Mechanisms are Best?

In order to answer this question, we need to decide what we mean when we say one mechanism is better than another. One might believe that mechanism A is better than mechanism B if and only if a majority would be willing to move [26] from B to A in all possible realizations of v. If so, then MR would be better than VCG. On the other hand, if one believes that mechanism A is better than mechanism B if and only if a unanimity would be willing to move from B to A in all possible realizations of v, then neither VCG nor MR is better than the other, since MR is never unanimously preferred to the VCG mechanism and there are always losers in a move from VCG to MR. Finally, if one believes that mechanism A is better than mechanism B if A always yields a higher aggregate pay-off in all possible realizations of v, then VCG would be (approximately) better than MR in large economies, since it is almost efficient.

In this chapter, I accept the implication of Arrow's theorem that there is no universally acceptable way to completely rank all mechanisms. Instead, I will look

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[24] By this I mean that $\lim_{x \to \infty} \int_x v^i f(v^i)dv^i = 0$ and $\lim_{x \to -\infty} \int^x v^i f(v^i)dv^i = 0$ where $f(\cdot)$ is the limiting density of types as $N \to \infty$.

[25] By this I mean that $F(\int v df(v)dv) \neq .5$: i.e. the median of the limiting distribution F is not equal to the mean.

[26] Those familiar with mechanism design will recognize that timing is important here. What the agents know when they make this decision is important. I am implicitly assuming here that they will know everything when they make their decision—all agents will have revealed their values. I am using an ex post analysis.

for mechanisms that are socially sensible. A minimal standard would be that it is common knowledge that there is no other mechanism such that everyone would be better off. To see how this works let us look at MR and VCG again. We do know, since neither chooses Pareto optimal allocations, that there are mechanisms that leave everyone better off. For example, let (y^{mr}, t^{mr}) be the MR process. Let (y^*, t^*) be any direct mechanism such that (a) $y^*(v) = 1$ iff $\sum(v^i - k) \geq 0$ (b) $\sum t^{*i}(v) = K y^*(v)$, and (c) $v^i y^*(v) - t^{*i}(v) \geq v^i y^{mr}(v) - t^{mr\,i}(v)$, with $>$ if possible. Since the allocations of MR are not efficient, we know such t^* exist. Therefore MR does not seem to survive our standard in very simple environments.[27] We can also do a similar thing for the VCG process so it also seems to fail our test.

So why don't we focus on efficient mechanisms like the (y^*, t^*) above instead of MR and VCG? Up to now we actually have been, by asking that our processes select Pareto optimal allocations in all environments. But, and this is a very important point, processes like (y^*, t^*) are not incentive compatible. Thus they are not implementable and so they really should not be considered to be viable challengers to either MR or VCG. We would be asking for too much.

I take a more forgiving approach and ask: what mechanisms are best among those that are incentive compatible? I will call a mechanism *incentive efficient* for a set of environments if it is incentive compatible and there is no other incentive compatible mechanism that is unanimously preferred to it over that set of environments.[28] Incentive efficient mechanisms are the best we can do within the reality of incentive constraints. If a mechanism is not incentive efficient, then one might expect it to be replaced by a more efficient, incentive compatible process. One would expect ubiquitous mechanisms, such as markets and voting, to have been able to survive such challenges. They would, therefore, be incentive efficient in exactly the sense I have chosen.

I can now rephrase my earlier question. Is either MR or VCG an incentive efficient mechanism over the simple public goods environments? Unfortunately I don't know the answer. In Roberts (1979), there is a characterization of the class of dominant strategy incentive compatible mechanisms for quasi-linear environments.[29] However, as far as I know, it remains an open problem to characterize the incentive efficient members of this class.

But it is too early to give up. It is time to try an indirect approach.

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[27] To emphasize that the efficiency concept is not that weak I note that (y^*, t^*) is also 'better' than MR under majority preference, unanimity preference, or aggregate value maximization.

[28] For those who are not familiar with mechanism design, Holmstrom and Myerson 1983 offer an excellent introduction to and discussion of efficient mechanisms in all their subtlety. They introduce the concept of incentive efficiency among many others also of importance.

Those familiar with mechanism design will recognize that in this section I am using the terms efficiency and incentive compatibility in the ex post sense, after everyone knows the entire vector v. I will take up the interim view in section 3.

[29] If $u^i = u^i(y, v^i) - t^i$, then the class of dominant strategy direct revelation mechanisms is given by (a) $y(m) = \arg\max F(y) + \sum_i k_i u^i(y, m^i)$ and $t^i(m) = \frac{1}{k_i}[\sum_{j \neq i} k_j u^i(y, m^j) + F(y)] + h_i(v_{-i})$ where $k \geq 0$ and F are arbitrary. Note that VCG belongs to this class by letting $k_i = 1$ for all i and $F(y) = 0$ for all y. I do not know what values of k and F yield the MR mechanism.

3 CAN WE SAY 'VOTING WORKS?'[30]

In this section, I raise the hurdle for a mechanism to be designated as incentive efficient by taking a Bayesian approach. I am going to broaden the class of mechanisms that will be considered to be incentive compatible and strengthen the criterion that must be satisfied for incentive efficiency. So it is now going to be harder for a mechanism to be incentive efficient. Such a mechanism will have to survive the challenge of more mechanisms and do it according to a stricter standard.

There is a price to pay for this strengthening. We will have to assume that there are prior beliefs, held by consumers, about the probability of any particular environment v. These beliefs will be common knowledge and, therefore, they can be used by the mechanism, creating even more challengers to potential incentive efficient mechanisms.

3.1 The Bayesian Approach

A very simple Bayes environment is described as follows. Begin with the very simple environment described in section 2.1 where $u^i = v^i y - t^i$. The v^i are assumed to be drawn independently and identically from a distribution $F(v)$ on $[a, b]$ and this is common knowledge to the consumers and the mechanism. The density associated with F will be denoted by $f(v)$.

In this world, I will look for incentive efficient mechanisms among those that are Bayesian incentive compatible.

Definition 1: *A direct mechanism, $[y(v), t(v)]$ is Bayesian incentive compatible (or interim incentive compatible) if and only if*

$$v^i \in \arg\max_{v'} \int \left[u^i(y(v/v'), v^i) - t^i(v/v') \right] dF(v|v^i)$$

where $(v/v') = (v^1, \ldots, v^{i-1}, v'^i, v^{i+1}, \ldots, v^N)$ and $dF = dF^1 \ldots dF^N$.

The use of the word 'interim' denotes that the evaluation of possible misreporting and other issues is done at the time at which all agents know their own type but do not yet know the types of the others. This is to be contrasted with the previous section in which such evaluations were done after all agents knew everyone's type. For that reason the analysis of the previous section is often referred to as ex post. If a mechanism is dominant strategy (or ex post) incentive compatible then it will be Bayesian (or interim) incentive compatible. So, for instance, both MR and VCG are Bayesian incentive compatible mechanisms. This means I am enlarging the class of incentive compatible mechanisms relative to the set of dominant strategy incentive compatible mechanisms of the previous section. This will make it harder for any

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[30] Much of this section is based on my joint work with Tom Palfrey. The paper that is most closely related is Ledyard and Palfrey 2002.

particular mechanism to emerge as incentive efficient since there are now more alternatives to compare it to.

I will look for mechanisms that are interim incentive efficient. These are mechanisms that are efficient when everyone knows their own type, v^i, but before anyone knows the other types.

Definition 2: *A mechanism $[y^*(v), t^*(v)]$ is interim incentive efficient iff there does not exist another mechanism $[y'(v), t'(v)]$ that is interim incentive compatible and for which*

$$\int u^i(y'(v), v^i) - t'^i(v)dF(v|v^i) \geq \int u^i(y^*(v), v^i) - t^{*i}(v)dF(v|v^i)$$

for all v^i, where the inequality is strict for a set of v^i of positive probability.[31]

There are at least three other equivalent definitions of interim incentive efficiency that can be helpful in understanding the concept. The first and most straightforward provides a simple reason why one should not expect interim incentive inefficient mechanisms to survive in practice.

Lemma 6: *A mechanism is interim incentive efficient iff it is common knowledge that there does not exist another interim incentive compatible mechanism that makes everybody better off.*

If a mechanism is not interim incentive efficient then it is common knowledge that everyone can be made better off with an alternative interim incentive compatible mechanism. So one should expect such an alternative to replace the current mechanism, even if all consumers know their own types.

The second definition is useful in searching for interim efficient mechanisms and can be derived from separating hyperplane theorems since the set of feasible and interim incentive compatible mechanisms is convex in the very simple Bayesian world.

Lemma 7: *A mechanism $[y^*(v), t^*(v)]$ is interim incentive efficient in the class of interim incentive compatible mechanisms iff there are $\lambda^i(v^i) > 0$ such that $[y^*(\cdot), t^*(\cdot)]$ solves*

$$\max \int \sum \lambda^i(v^i)[v^i y(v) - t^i(v)]dF(v)$$

subject to

$$\sum t^i(v) \geq Ky(v)$$

and

$$v^i \in \arg\max \int v^i y(v/v'^i) - t^i(v/v'^i)dF(v|v^i).$$

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[31] It is important to remember that I continue to forego any individual rationality requirements. In Ledyard and Palfrey 2005 we considered interim efficient mechanisms that are individually rational.

The third alternative definition asserts that a mechanism is interim incentive efficient if and only if it is ex ante incentive efficient for all affine transformation of the utility functions.[32] It is this fact that convinces me, at least for a Bayesian analysis, that interim incentive efficiency is the right concept to use when trying to identify mechanisms that will survive and be used.

3.2 A Bayesian Characterization of Interim Incentive Efficient Mechanisms

In Ledyard and Palfrey (1999), we were able to characterize the class of interim incentive efficient mechanisms for the very simple Bayesian environment. In these, consumers announce their values, y is chosen according to a virtual cost–benefit rule, and taxes are computed using a rule discovered by d'Aspremont and Gerard-Varet (1979). We called these mechanisms virtual cost–benefit mechanisms.

Definition 3: *The VCB (virtual cost–benefit) mechanism, for given functions $\lambda^i(v^i)$, is given by $m^i \in M^i = V^i$, an output rule*

$$y^*(v) = 1 \text{ if } \sum_i w^i(v^i) \geq K$$

$$y^*(v) = 0 \text{ otherwise,}$$

where

$$w^i(v^i) = v^i + \{[F^i(v^i)) - \Lambda^i(v^i)]/f^i(v^i)\},$$

$$\Lambda^i(v^i) = \int^{v^i} \lambda^i(s)ds$$

and a tax rule

$$t^{*i}(v) = ky^*(v) - [T^i(v^i) - Q^i(v^i)] + \left(\frac{1}{N-1}\right)\left\{\sum_{j \neq i}[T^j(v^j) - Q^j(v^j)]\right\}.$$

where

$$Q^i(v^i) = \int^{v^i} y^*(x)dF(x|v^i)$$

and

$$T^i(v^i) = \int^{v^i} s\,dQ^i(s).$$

[32] 'Ex ante' refers to the analysis that is done prior to anyone knowing anything other than the common knowledge. A mechanism is ex ante efficient iff there are $\lambda^i > 0$ such that $[y^*(\cdot), t^*(\cdot)]$ solves $\max \int \sum \lambda^i[v^i y(v) - t^i(v)]dF(v)$ subject to feasibility and incentive compatibility. Compare this with the definition of interim efficiency to see that the difference is that the λ^i do not depend on the v^i. An affine transformation of utilities for each v^i yields $u'^i = a^i(v^i)[v^i y(v) - t^i(v)] + b^i(v^i)$. This does not change either the feasibility or incentive constraints. But it does change the objective function to $\int \sum \lambda^i a^i(v^i)[v^i y(v) - t^i(v)]dF(v)$. So, letting $\lambda^i(v^i) = \lambda^i a^i(v^i)$, it is easy to see that a mechanism is ex ante efficient for all affine transformation of utilities iff it is interim efficient.

Myerson (1981) called the w^i virtual valuations which is why we refer to this as a virtual cost–benefit rule—the good is produced if and only if the virtual benefits outweigh the costs. The key result from Ledyard and Palfrey (1999) is

Lemma 8: *All interim incentive efficient mechanisms in the very simple Bayesian environment are VCB mechanisms for some λ.*

There are a couple of things to note. First, all interim incentive efficient mechanisms are resource efficient. That is, $\sum t^i(v) = K y(v)$ for all v. One important implication of this is that VCG mechanisms will generally not be interim incentive efficient since they rarely satisfy resource efficiency. Second, output efficiency is neither necessary nor sufficient for interim incentive efficient mechanisms.[33] In fact, interim incentive efficiency implies output inefficiency for almost all weights λ. The only case in which output efficiency holds is for $\lambda^i(v^i) = 1$. In this singular case, the VCB mechanism selects Pareto optimal allocations. On the other hand, if λ is increasing in v, so that high-value types are weighted more heavily than low-value types, then the interim incentive efficient mechanisms for those λ will require the public good to be produced more often than is (ex post) efficient.[34] If λ is decreasing in v then interim-incentive-efficiency will require under-production of the public good.

At first glance the characterization does not bode well for the sought-after conclusion that 'Voting Works.' VCB mechanisms do not look very much like voting mechanisms. Further, since majority rule is interim incentive compatible and is not a VCB mechanism, it is not interim incentive efficient. I address this problem in two ways. I first broaden the class of voting mechanisms beyond simple MR, and then, as is done with markets for private goods, turn to large economies.

3.3 'Voting Works' in Large Bayesian Environments

In order to more easily explain why voting seems to work in large environments, we begin by looking at what the interim efficient VCB mechanisms look like in large economies. As $N \to \infty$, feasibility, quasi-linearity, and symmetry will imply that i's effect on i's conditional expected output, $dQ^i(v^i)/dv^i$, goes to zero. If that is true, then incentive compatibility requires that i's effect on i's conditional expected taxes, $dT^i(v^i)/dv^i$, should also go to zero.[35] This means that each individual's taxes must begin to look like per capita taxes.[36] It follows that, in large economies, all interim-incentive-compatible taxes are approximately per capita taxes.[37] This means that

[33]   Over- and under-production are used, in VCB mechanisms, to relax the incentive compatibility constraints at a lower cost than using taxes as is done in VCG mechanisms.

[34]   That means there will be times it is produced even if $\sum v^i - K < 0$.

[35]   Incentive compatibility requires that $v^i dQ^i(v^i)/dv^i = dT^i(v^i)/dv^i$. So the right-hand side $\to 0$ as $N \to \infty$. Otherwise i will try to avoid taxes by misreporting her type.

[36] $T^i(v^i) \to kQ^i(v^i)$ and $t^i(v) \to ky(v)$ for all i as $N \to \infty$.

[37]   This statement about the necessity of approximately constant per capita taxes for incentive compatibility in large environments holds considerably more broadly. It is true in public goods environments in which the public good is multidimensional and in which the types of the consumers are multidimensional.

voting procedures, if they can get the output decision right, have a chance to perform as well as the interim incentive efficient VCB mechanisms. There is a class that does.

Consider the class of voting processes, which I call q-referenda.[38] In these voting processes, each agent is asked to vote yes or no for the public good. The good is produced if and only if the proportion of yes votes is greater than or equal to q. Each agent pays the per capita cost of the public good.

Definition 4: [39]*A class of voting processes called q-referenda is given by: $m^i \in M^i = \{0, 1\}$, where 0 is a no vote and 1 is a yes vote, an outcome rule*

$$y(m) = 1 \text{ iff } \sum_i m^i \geq Nq,$$

and a taxation rule

$$t^i(m) = ky(m).$$

It is a dominant strategy to vote yes if and only if your value for the good is at least as much as the per capita cost of the good. So, all q-referenda are incentive compatible in both the dominant (ex post) and Bayesian (interim) senses. It is easy to show that q-referenda do not produce Pareto optimal allocations: they are resource efficient but not output efficient. But in large economies they begin to look very good. In particular, as $N \to \infty$, the set of q-referenda is virtually equivalent to the set of interim efficient mechanisms.

Using results from Ledyard and Palfrey (2002), one can show that for every λ, there is a q such that, as $N \to \infty$, the expected value of the q-referendum converges to the expected value of that λ-VCB mechanism.

Theorem 1: *Let (y^λ, t^λ) be the output and tax rules for the λ-VCB mechanism. Let (y^q, t^q) be the output and tax rules for the q-referendum. For every λ with $\int \lambda^i dF^i(v^i) > 0$ for all i, there is a $q \in [0, 1]$ such that as $N \to \infty$*

$$\int \sum \lambda^i(v^i)[v^i y^q(v) - t^{qi}(v)]dF \to \int \sum \lambda^i(v^i)[v^i y^\lambda(v) - t^{\lambda i}(v)]dF.$$

Proof: The q that works solves $qE\{\lambda v | v > k\} + (1 - q)E\{\lambda v | v < k\} = k$.

A converse to this result also holds. For every q-referendum there is a λ-VCB mechanism that approximates it. This means that, in large economies, q mechanisms are approximately interim incentive efficient and any interim incentive efficient mechanism can be approximated with a q mechanism. In particular, in large economies, to an appropriate approximation, no mechanism—be it VCG or VCB—can replace a q-referendum and make everyone better off. On these grounds I would tentatively claim that 'Voting Works' in an interim sense in a Bayesian context.[40]

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[38] $q \in [0, 1]$ and represents the required plurality for acceptance of the referendum proposition that 'the public good should be produced and financed with per capita taxes.' If the required plurality is not achieved, 'the public good is not produced and no one pays any taxes.'

[39] A direct mechanism version of q-referenda is given by: $m^i \in V^i$, an outcome rule $y(v) = 1$ iff $\#\{v^i | v^i \geq k\} \geq Nq$, and a taxation rule $t^i(v) = ky(v)$.

[40] It is shown in Malath and Postlewaite 1990 that if we impose a requirement of interim individual rationality then as $n \to \infty$ the only feasible, incentive compatible, interim individually rational

3.4 'Voting Works' in Large Environments

But we can actually say more by recognizing two facts: (1) q-referenda are incentive compatible in dominant strategies and (2) any interim efficient mechanism is also an efficient mechanism. Using the first fact, if D is the set of all dominant strategy mechanisms and (y^q, t^q) is a q referendum, then

$$\int \sum \lambda^i(v^i)[v^i y^q(v) - t^{qi}(v)]dF \leq \max_D \int \sum \lambda^i(v^i)[v^i y(v) - t^i(v)]dF.$$

and using the second fact, since $D \subset B$ where B is the set of all Bayesian incentive compatible mechanisms

$$\max_D \int \sum \lambda^i(v^i)[v^i y(v) - t^i(v)]dF. \leq \max_B \int \sum \lambda^i(v^i)[v^i y(v) - t^i(v)]dF.$$

where

$$\max_B \int \sum \lambda^i(v^i)[v^i y(v) - t^i(v)]dF = \int \sum \lambda^i(v^i)[v^i y^\lambda(v) - t^{\lambda i}(v)]dF$$

and y^λ, t^λ is the VCB mechanism for λ. Now stringing all of these together, it follows that, for the appropriate q, as $N \to \infty$ all of these expressions approach each other in value, because the largest and smallest do by Theorem 1. This means in particular that as $N \to \infty$

$$\int \sum \lambda^i(v^i)[v^i y^q(v) - t^{qi}(v)]dF \to \max_D \int \sum \lambda^i(v^i)[v^i y(v) - t^i(v)]dF.$$

and so the q-referendum is approximately incentive efficient in large economies.[41]

I would claim that at this point we have ample grounds for saying that, if we forgo any requirement for individual rationality, 'Voting Works' in large simple public goods environments.

3.5 Some Doubts

There are several reasons to pause here before declaring total victory.

In large economies, not only is reporting truth an incentive compatible behavior, so is almost anything else. If $dQ^i(v^i)/dv^i \approx 0$, and $dT^i(v^i) \approx 0$, then my report has virtually no effect on my utility. So almost any behavior, $m(v)$, will be incentive compatible in q-referenda. This means there are multiple equilibria in the mechanism which in turn means actual behavior is a bit unpredictable. This is a problem with many mechanisms. For example, the dynamic voluntary contributions game has

mechanisms do not produce the public good. We already know that the only feasible, incentive compatible mechanisms as $n \to \infty$ look like majority rule. And majority rule violates individual rationality unless unanimity is required, in which case nothing is ever produced.

[41] A warning is appropriate here. This does not mean that q-referenda are approximately equivalent to the set of incentive efficient mechanims. There may be incentive efficient mechanisms that cannot be approximated by a q-referendum.

equilibria that are Pareto optimal (see Marx and Matthews 2000), so it is an incentive compatible and efficient mechanism for public goods. But that mechanism also has multiple equilibria.[42]

Another reason to pause is that in large environments, as we have defined them, it is possible for command mechanisms to be interim incentive efficient. By a command mechanism, I mean a mechanism which does not ask for any information from consumers. Such mechanisms are really simple. Consider the following mechanism: $y = 1$ iff $\int \lambda(v) v \, dF(v) \geq k$ and $t^i = ky$. That is, if the prior beliefs are such that the expected weighted value of the public good is bigger than the per capita cost then produce the good. As $N \to \infty$, since the v^i are independently and identically distributed, the law of large numbers means that the probability that $\{y = 1$ iff $\sum \lambda^i(v^i)[(v^i - k)] \geq 0\} \to 1$. It is trivially incentive compatible. That is, the command mechanism is almost interim incentive efficient. The force of this observation can be blunted by considering a more complex environment in which the types, v^i, are correlated. For example, let $v^i = r^i + c$ for all i where r^i is distributed independently and identically according to $F(r)$ and c is distributed according to $G(c)$. Then unilateral mechanisms will not be interim incentive efficient, even approximately, but Theorem 1 remains valid. So if values are correlated q-referenda are almost interim incentive efficient but unilateral mechanisms are not.

Another reason to hesitate is that we do not know whether the results so far can be extended to simple environments with a variable quantity public good.[43] Ledyard and Palfrey (1999) provide the appropriate characterization of the VCB mechanisms for the case of a variable public good when the utility functions are linear in v^i. It is an open question whether any voting processes provide an approximation to these generalized VCBs.

3.6 Multidimensional Issues Spaces

Finally there is a question of what to do in multidimensional public goods problems. In the multidimensional policy spaces that arise when there is more than one public good or when the tax functions are up for grabs, there is rarely a median voter. This causes serious problems for many models of voting such as simple majority rule processes, as was illustrated earlier in Example 15. But one can still find voting processes that 'work.'[44]

Using techniques developed in Berliant and Ledyard (2004) and Ledyard (1984), one can find a mechanism which is interim incentive efficient in a world with multiple public goods, non-linear tax schedules of income, and without quasi-linear utility functions. In this process, two candidates vie for election. Each wants to maximize

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[42] I thank Steve Matthews for reminding me about the issues surrounding multiple equilibria. I do not really confront this fully here.

[43] This is the case in which $u^i = g(y, v^i)$.

[44] My space is limited and so I have to forgo an extensive discussion at this point and just make the claim. The interested reader can pursue the details in Ledyard 2005.

their probability of winning. It is costly to vote and that cost varies across consumers. A voter abstains if the expected benefits of voting do not outweigh the cost of voting. Voters take candidate positions as given and then turn out and vote given rational expectations about other voters. Candidates, knowing how voters behave, choose equilibrium platforms. Any equilibrium of the two-candidate election will be interim incentive efficient in the space of implementable platforms.[45]

4 AN AGGRESSIVE CONCLUSION
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In this chapter, I have explored whether there were any conditions under which voting processes are good mechanisms for the allocation of and taxing for public goods. I was looking for something that compared to the central claim of welfare economics that 'Markets Work' for the allocation of private goods. In that literature, it is demonstrated that, in large economies, there are market processes that are incentive compatible and almost efficient[46] and there are market processes that are efficient and almost incentive compatible.[47] Even though one cannot find mechanisms that are simultaneously incentive compatible and efficient in large economies (or small), these results are generally viewed as sufficient to provide a theoretical rationale for the ubiquitous nature of markets.

In this chapter I find similar results for voting and public goods. In large economies, voting is shown to be incentive compatible and almost incentive efficient, where the latter means that it is common knowledge that there is no other incentive compatible mechanism such that everyone is better off by more than a very small amount.

In large economies, if we dispense with the individual rationality constraint, 'Voting Works.'

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BERLIANT, M., AND LEDYARD, J. 2004. A direct approach to optimal income taxation. Manuscript.

BEWLEY, T. 1973. Edgeworth's conjecture. *Econometrica*, 41: 425–54.

[45] It would be nice to also have a result that characterized all interim efficient mechanisms in this world. I do not have such a theorem at this time.

[46] The examples can be found in McAfee 1992 and Gresik and Satterthwaite 1989.

[47] The canonical example is competitive equilibrium with price-taking behavior.

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