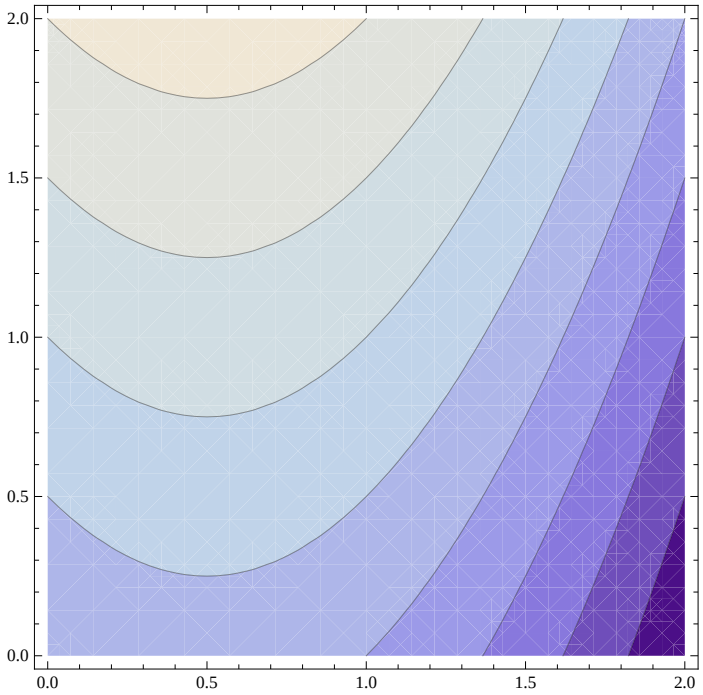
The Lazy Housekeepers' Problem

The Symmetric Case

Let x be the amount of housekeeping effort that Alice supplies and y the amount that Bob supplies. Alice has utility function U_A(x,y)=x+y-x^2 and Bob has utility function U_B(x,y)=x+y-y^2. If we draw a box with x on the horizontal axis and y on the vertical axis, Alice's indifference curves are shown as the U-shaped curves below. Regardless os what Bob does, she would prefefr to do 1/2 unit of housework. But the more housework Bob does, the happier she is.

In[428]:=
```
aplot = ContourPlot[x + y - x^2, {x, 0, 2}, {y, 0, 2}, Axes → True]
```

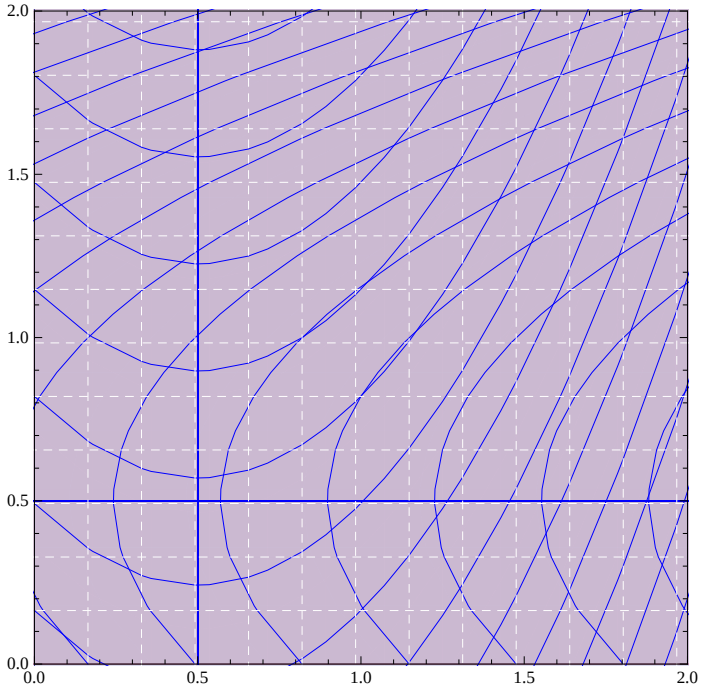
Out[428]=



We can draw Bob's indifference curvves on the same axes. This is shown in the fiture below. I used the command Parametric plot to draw a bunch of indifference curves each of which holds u constant at some value between -4 and 4 and letting x vary from 0 to 10. I restrict the range of x's and y's that are displayed to values between 0 and 2.

In[453]:=
```
boxsymplot =
 ParametricPlot[{{x, u + x^2 - x}, {u + x^2 - x, x}, {0.50, x}, {x, 0.5}}, {u, -10, 10},
  {x, 0, 10}, AspectRatio → 1 / 1, PlotRange → {{0, 2}, {0, 2}}, Background → White, Mesh → 60
  , MeshStyle → {Directive[Blue], Directive[White, Dashed]}]
```

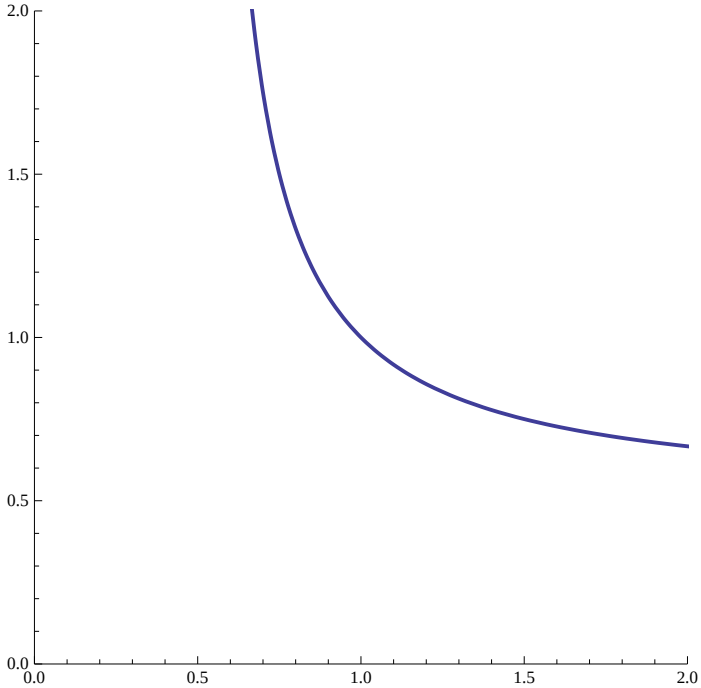
Out[453]=



Next I draw the contract curve. As shown in our notes, this has the equation y=x/^2x-1),

In[439]:=
```
ContCurveSym = ParametricPlot[{x, x / (2 * x - 1)}, {x, .51, 10},
  AspectRatio → 1 / 1, PlotRange → {{0, 2}, {0, 2}}, PlotStyle → Thick ]
```

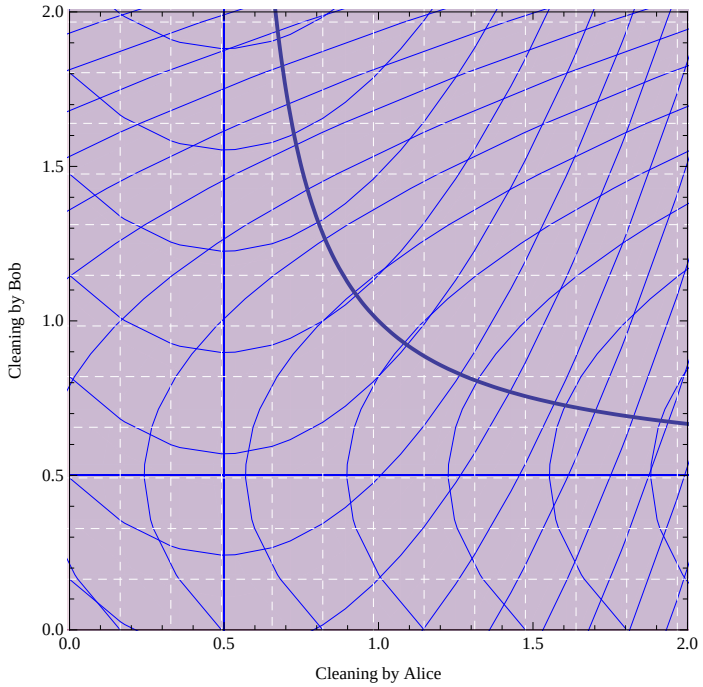
Out[439]=



Now we display the contract curve on the labelled Edgeworth box.

In[454]:=
```
Show[boxsymplot, ContCurveSym, Frame → True,
 FrameLabel -> {"Cleaning by Alice", "Cleaning by Bob"}]
```

Out[454]=

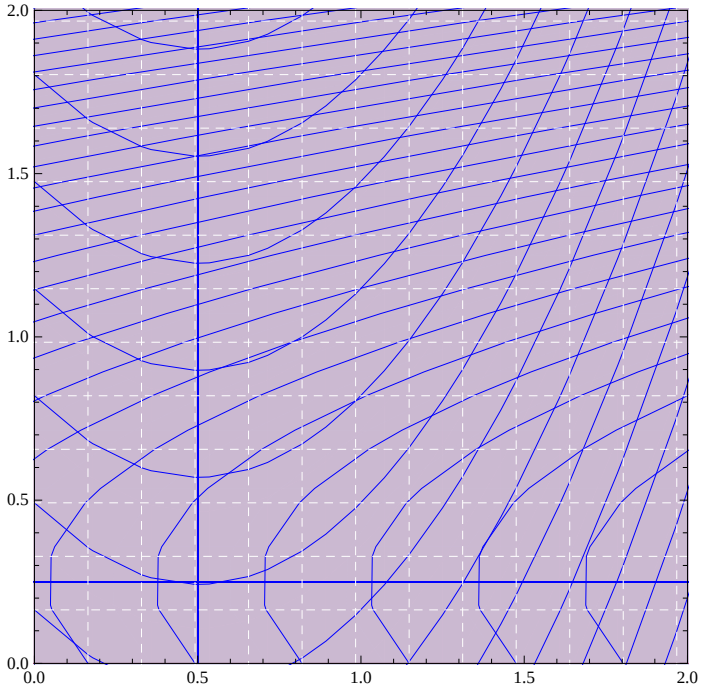


Bob hates housekeeping more than Alice

Let utilities be u_A=x+y-x^2 and u_B=x+y-2y^2. We first plot the Edgeworth box. Then we add the contract curve and display them both together.

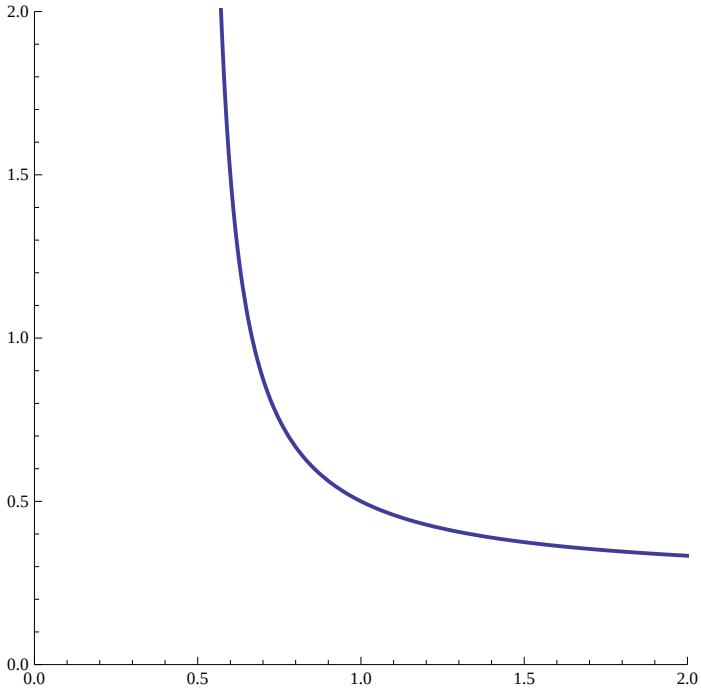
In[452]:=
```
box12plot = ParametricPlot[{{x, u + x^2 - x}, {u + 2 * x^2 - x, x}, {0.50, x}, {x, 0.25}},
  {u, -10, 10}, {x, 0, 10}, AspectRatio → 1 / 1, PlotRange → {{0, 2}, {0, 2}},
  Mesh → 60, MeshStyle → {Directive[Blue], Directive[White, Dashed]}]
```

Out[452]=



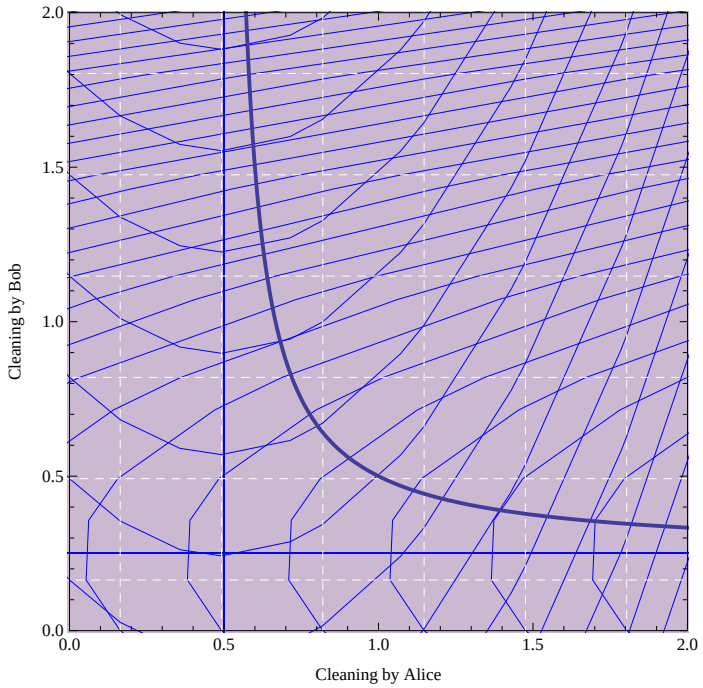
In[442]:=
```
contcurve12 = ParametricPlot[{x, x / (2 * (2 * x - 1))}, {x, .51, 10},
  AspectRatio → 1 / 1, PlotRange → {{0, 2}, {0, 2}}, PlotStyle → Thick ]
```

Out[442]=



In[443]:=
```
Show[box12plot, contcurve12, Frame → True,
 FrameLabel -> {"Cleaning by Alice", "Cleaning by Bob"}]
```

Out[443]=



We construct the utility possibility frontier for the

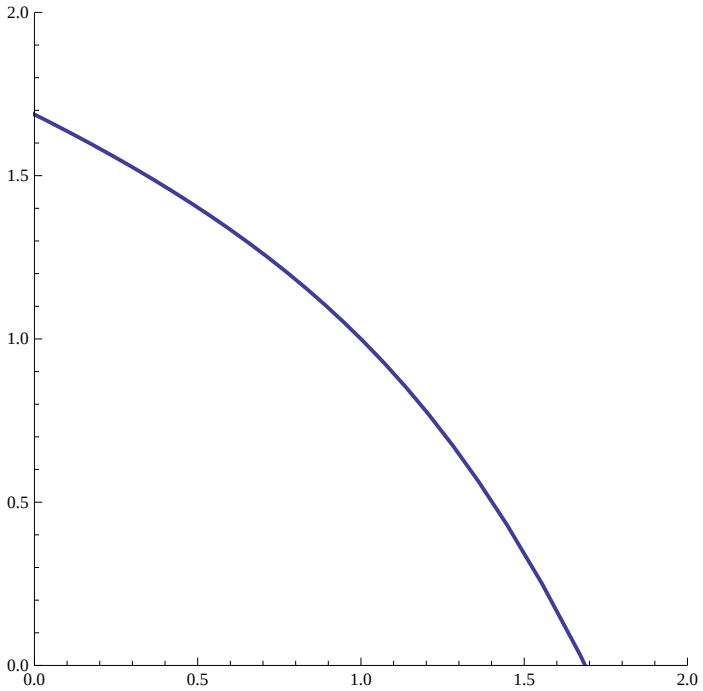
Utility possibility frontiers

We construct the utility possibility frontier for the symmetric and asymmetric cases. To do this, I use ParametricPlot, varying x from 1/2 to 2, using the efficiency conditions to determine y from x and then plotting the utilities for each of the two people as x is varied while y moves with x according to the efficiency condition. In the symmetric the symmetri case, we have
y=x/(2x-1) along the contract curve. In the asymmetric case, y=x/(4x-2) along this curve.

■ Utility possibility frontier: Symmetric Case

In[445]:=
```
uposssym = ParametricPlot[{x + (x / (2 * x - 1)) - x^2, x + (x / (2 * x - 1)) - (x^2) / (2 * x - 1)^2},
  {x, .51, 2}, PlotRange → {{0, 2}, {0, 2}}, AspectRatio → 1 / 1 , PlotStyle → Thick]
```

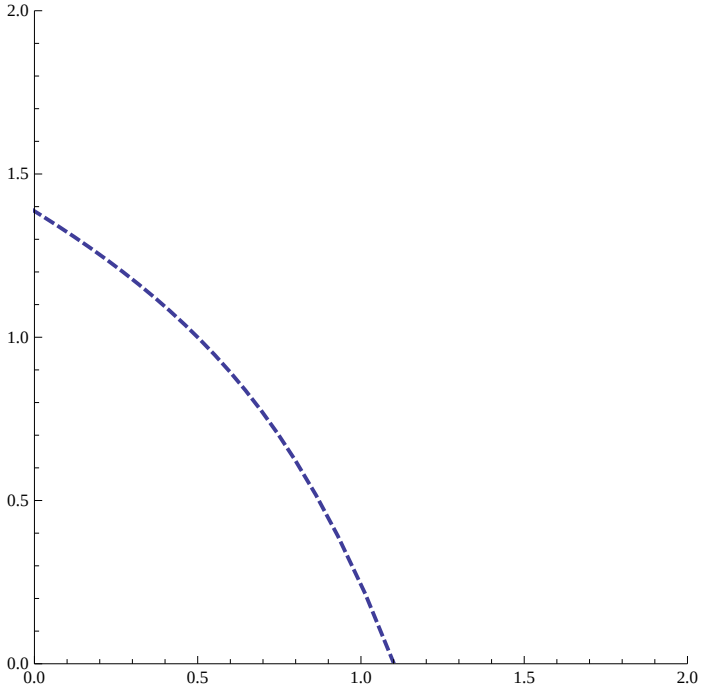
Out[445]=



■ Utility possibility frontier: Case where Bob hates housekeeping More

In[446]:=
```
uposs12 = ParametricPlot[
  {x + (x / (4 * x - 2)) - x^2, x + (x / (4 x - 2)) - 2 * ((x^2) / (4 x - 2)^2)}, {x, .51, 2},
  PlotRange → {{0, 2}, {0, 2}}, AspectRatio → 1 / 1 , PlotStyle → Directive[Thick, Dashed]]
```

Out[446]=



■ Both cases together

In[447]:=
```
Show[uposssym, uposs12, ListLinePlot[{{2.01, 0}, {0, 2.01}}],
 ListLinePlot[{{1.5, 0}, {0, 1.5}}], ListPlot[{{1, 1}, {1 / 2, 1}}, PlotMarkers → Automatic],
 Frame → True, FrameLabel → {"Utility for Alice", "Utility for  Bob"}]
```

Out[447]=