Homework Problems on Expected Utility
Fall 2009, Econ 210A UCSB

1. Randy Variable is an expected utility maximizer with a von Neumann Morgenstern utility function $v(x) = x^{1/2}$. He has a wealth of $\$99,000$. His shady brother-in-law has given him inside information on the outcome of an upcoming sports event. On the basis of this information, Randy believes that the probability that Team A will defeat Team B is 2/3, despite the fact that Team B is two-to-one favorite to win among the betting community. It is possible for Randy to bet as much money as he wishes on Team A to win at the prevailing odds. That is, for every dollar that he bets, he will get back \$2 if Team A wins and will get back nothing if Team A does not win.

a. Let x_A be the contingent commodity "wealth if Team A wins" and let x_B be "wealth if Team B wins." Write a budget equation expressing the combinations of x_A and x_B that Randy can afford by betting some amount of money on Team A. Draw a graph to show all of these combinations.
B. If Randy bets so as to maximize his expected utility, what will be his wealth if Team A wins?

What will be his wealth if Team B wins?

C. Peter Punter has a von Neumann Morgenstern utility function $v(x) = -x^{-1}$ and he also has a wealth of $\$99,000$. He has the same shady brother-in-law as Randy and the same inside information on the outcome of an upcoming sports event. Peter also believes that the probability that Team A will defeat Team B is 2/3, despite the fact that Team B is two-to-one favorite to win among the betting community. It is likewise possible for Peter to make bets such that for every dollar that he bets, he will get back \$2 if Team A wins and will get back nothing if Team A does not win. If Peter places his bets to maximize his expected utility, what will be his wealth if Team A loses?

What will be his wealth if Team A wins?

2) Dracula, the mortgage broker, is an expected utility maximizer, with von Neumann-Morgenstern utility function

$$u(x) = \frac{1}{2}\sqrt{x}.$$

A) Dracula currently holds a portfolio of subprime mortgages, all in the same town. If the local economy goes bad, these mortgages will be worthless and his wealth will be zero. If the local economy does not go bad, his wealth will be \$1,000,000. The probability that the local economy will go bad is 1/10. Calculate the certainty equivalent of Dracula's current holdings.

B) Suppose that Dracula can buy mortgage insurance such that he will pay \$X to the insurer if the mortgages do not fail and he will receive a payment

of \$1,000,000-\$X from the insurer if the mortgages fail. What is the largest amount \$X that Dracula would be willing to pay for this insurance.

C) What is the largest amount that Dracula would be willing to pay for the insurance if his von Neumann-Morgenstern utility function were linear in his wealth?

D) Suppose that in addition to his risky mortgage portfolio, Dracula has safe assets that are worth \$V, no matter what happens. Write an expression for the certainty equivalent of Dracula's assets (as a function of V), assuming he does not buy insurance. Write an expression for the largest amount \$X that Dracula would be willing to pay for the insurance described in Part B (as a function of V).

3) A poor fellow somehow acquires a lottery ticket that with equal probabilities will be worth 20,000 ducats or 0. His von Neumann-Morgenstern utility function is the natural log of his wealth. In the absence of the lottery ticket, his wealth is X. He is offered an opportunity to sell this ticket for 9000 ducats. How large does X need to be in order to refuse this offer? (You will probably want to use a calculator.)

A rich fellow whose von Neumann-Morgenstern utility function is the natural log of his wealth is considering buying this lottery ticket from the poor fellow for a price of 9,000 ducats. How wealthy does the rich fellow have to be so that he would choose to buy it?