Name ______________________

Midterm Examination: Economics 210A

October 16, 2015

Question 1.) In their 1972 book, Economic Theory of Teams, *Jacob Marschak and Roy Radner describe a team theory problem as one in which n players all want to maximize the same objective function. Each player chooses an action x_i from a set X_i and each player gets a payoff equal to $F(x_1, \dots, x_n)$. There is no "boss" to coordinate their activities. Marschak and Radner define an outcome $\bar{x} = (\bar{x}_1, \dots, \bar{x}_n)$ to be* Person-by-person satisfactory *if each person is taking the action that maximizes F, given what the other players are doing. This means that for all players i:*

$$F(\bar{x}_1, \dots, \bar{x}_i \dots, \bar{x}_n) \geq F(\bar{x}_1, \dots, x_i, \dots, \bar{x}_n)$$

for all $x_i \in X$.

If $\bar{x}_i$ is in the interior of X_i for all i and if $\bar{x}$ is person-by-person satisfactory, what must be true of the first and second order partial derivatives of F?

Answer At a person by person satisfactory outcome, each player is doing the best thing for the team *given what the others are doing.* The necessary conditions for an interior maximum for a single player are that $F_i(\bar{x}) = 0$ and $F_{ii}(\bar{x}) \leq 0$.

For now let us consider two-person teams. Assuming that F is twice continuously differentiable, determine whether each of the following three statements is true or false. If it is true, sketch a proof. If it is false, provide a counterexample.

Claim 1) *If $(\bar{x}_1, \bar{x}_2)$ maximizes $F(x_1, x_2)$ on the set on (X_1, X_2), then outcome $(\bar{x}_1, \bar{x}_2)$ must be person-by-person satisfactory.*

Answer True. Proof: If $(\bar{x}_1, \bar{x}_2)$ maximizes $F(x_1, x_2)$ on the set on (X_1, X_2) then it must be that $F(\bar{x}_1, \bar{x}_2) \geq F(x_1, x_2)$ for all $(x_1, x_2) \in (X_1, X_2)$, so in particular it must be that $F(\bar{x}_1, \bar{x}_2) \geq F(x_1, \bar{x}_2)$ for all $x_1 \in X_1$ and it must also be that $F(\bar{x}_1, \bar{x}_2) \geq F(\bar{x}_1, x_2)$ for all $x_2 \in X_2$. But this tells us that if $(\bar{x}_1, \bar{x}_2)$ maximizes $F(x_1, x_2)$ on the set on (X_1, X_2) , $(\bar{x}_1, \bar{x}_2)$ must be person-by-person satisfactory.

Claim 2) *If outcome $(\bar{x}_1, \bar{x}_2)$ is person-by-person satisfactory, it must be that $(\bar{x}_1, \bar{x}_2)$ maximizes $F(x_1, x_2)$ on (X_1, X_2).*

Answer False. When $(\bar{x}_1, \bar{x}_2)$ is person-by-person satisfactory it is not possible for one person to increase the payoff by changing his action, but it might be possible for both. To produce a counterexample, just find a function $F(x_1, x_2)$ that satisfies the necessary calculus conditions for person-by-person satisfactoriness, but where the Hessian matrix is not negative semi-definite. We worked with just such an example in class. It was

$$F(x_1, x_2) = x_1 + x_2 - \frac{1}{2}(x_1^2 + x_2^2) + cx_1x_2$$

with $c > 1$.
Claim 3) *If F is a concave function, then if an outcome is person-by-person satisfactory and x_i is in the interior of X_i for $i = 1, 2$, it must be that $(\bar{x}_1, \bar{x}_2)$ maximizes $F(x_1, x_2)$ on (X_1, X_2).*

AnswerThis one is true. If F is a concave function, then its Hessian is negative semi-definite. If an outcome is person by person satisfactory, its partial derivatives are zero. If F is a concave function and its partial derivatives are all 0 at $\bar{x}$, then $\bar{x}$ must be a maximum.

Question 2.) *A consumer has preferences on $X \subset \Re^2_+$ which are defined in the following way. There is a real-valued continuous function u with domain X such that for all x and y in X, $x \succeq y$ if and only if $u(y) - u(x) \leq 1$.*

A) Where the relations $\succ$ and $\sim$ are defined from $\succeq$ in the usual ways, describe the relations $x \succ y$ and $x \sim y$ in terms of inequalities involving $u(x)$ and $u(y)$.

Answer $x \succ y$ if $u(x) - u(y) > 1$. $x \sim y$ if $|u(x) - u(y)| \leq 1$

B) Determine whether each of the following three statements is true or false. If it is false show a counterexample. If it is true, prove it.

Claim 1) *The relation $\succeq$ is transitive.*

Answer

False: Let $u(x) = 3$, $u(y) = 2$ and $u(z) = 1$. Then $z \succeq y$, $y \succeq x$, but NOT $z \succeq x$.

Claim 2) The relation $\succ$ is transitive.

Answer True. If $x \succ y$ and $y \succ z$, then $u(x) - u(y) > 1$ and $u(y) - u(z) > 1$. Therefore $u(x) - u(z) = (u(x) - u(y)) + (u(y) - u(z))$
Claim 3) The relation $\succeq$ is continous.

C) Suppose that $\succeq$ is described as above, where $u(x_1, x_2) = x_1 x_2$. Let B be the set of things the consumer can afford with income 10 and where the price of good 1 is 1 and the price of good 2 is 2. Is $c(B)$ non-empty? Sketch the set $c(B)$. Describe this set, using two inequalities.