Name ______________________________

Final Exam, Economics 210A, December 2012

There are 8 questions. Answer as many as you can... Good luck!

1) Let S and T be convex sets in Euclidean n space. Let $S+T$ be the set $\{x|x = s+t$ for some $s \in S$ and some $t \in T\}$. For any real number a, and any set S, let $aS = \{x|x = as$ for some $s \in S\}$ For each of the following statements, tell me whether it is true of false. If true, prove it. If false, give a counterexample.

A) $S+T$ is a convex set
B) aS is a convex set if a is a positive real number.
C) aS is a convex set if a is any real number.

2) A firm has production function $f(x_1, x_2) = (x_1 - a)^{1/4}(x_2 - b)^{3/4}$, where a and b are non-negative constants, and x_1 and x_2 are input quantities. The firm can buy factor 1 at price w_1 per unit and factor 2 at price w_2 per unit.

A) What is this firm's cost function if $a = b = 0$?

B) What is this firm's cost function for general a and b?

C)What is this firm's conditional factor demand function for factor 1, for general a and b?

3) Jake has expenditure function $e(p_1, p_2, u) = u\sqrt{p_1 p_2}$. Jane has utility function $u(x_1, x_2) = ax_1^3 x_2^b + c$.

A) Find the Marshallian demand functions for each of the two consumers.

B) For what values of the parameters a, b, and c will aggregate demand be independent of the distribution of income?

4) Petraus is an expected utility maximizer with von Neumann Morgenstern utility function $u(w) = \ln w$. He is offered a chance to bet on the outcome of a flip of a coin that he believes will come up heads with probability π. If he bets x, he will have wealth $w + y$ if the coin comes up heads and $w - x$ if it comes up tails.

A) If $y = x$, solve for the optimal amount x for him to bet as a function of π and w.

b) If $y = x - c$ for some $c > 0$, solve for the optimal amount for him to bet as a function of π and w.

5) A pure exchange economy has 5,000 consumers and 3 goods X, Y, and Z. Every consumer i in the economy has utility function

$$U_i(x_i, y_i, z_i) = \left(x_i^{1/2} + y_i^{1/2} + z_i^{1/2}\right)^2,$$

where x_i, y_i, and z_i are i's consumption of X, Y, and Z, respectively. There are 3 types of consumers. There are 2,000 Type 1 consumers and each has an initial endowment of 4 units of good x and 0 units of goods y and z. There are 1000 Type 2 consumers and each has an initial endowment of 2 units of good y and 0 units of goods x and z. There are 2000 Type 3 consumers and each has an initial endowment of 1 unit of good z and 0 units of goods x and y.

A) Let Good X be the *numeraire* good and find competitive equilibrium prices for Goods Y and Z.

B) Compare the "income" (value of initial endowment) of a Type 1 consumer with that of a Type 2 consumer and with that of a Type 3 consumer. What is the total value of initial endowments at competitive equilibrium prices?

C) In competitive equilibrium, how much Good X does a Type 1 consume? How much Good X does a Type 2 consume? How much Good X does a Type 3 consume?

D) A natural disaster strikes the suppliers of Good 1 and reduces the initial endowment of each Type 1 consumer from 4 units of good X to 1 unit of good X. What are the prices in the new competitive equilibrium? **E** In the new equilibrium, compare the value of the endowment of a Type 1 consumer with that of a Type 2 consumer and that of a Type 3 consumer.

6) Consider an economy in which endowments are as in the previous problem, before the natural disaster, but utility functions are

$$U_i(x_i, y_i, z_i) = \left(x_i^{-1/2} + y_i^{-1/2} + z_i^{-1/2}\right)^{-2}.$$

A) Find the competitive prices for this economy.

B) Compare the "income" (value of initial endowment) of a Type 1 consumer with that of a Type 2 consumer and with that of a Type 3 consumer.

C) Suppose that in this economy a natural disaster strikes the suppliers of Good 1 and reduces the initial endowment of each Type 1 consumer from 4 units of good X to 1 unit of good X. What are the prices in the new competitive equilibrium? In the new equilibrium, compare the value of the endowment of a Type 1 consumer with that of a Type 2 consumer and that of a Type 3 consumer.

D) (For extra credit) In this case would Type 1's benefit from the disaster? Can you say anything more general about when a type would gain (lose) if the endowments of everyone of that type were to increase (decrease)?

7) Consider a three-dimensional surface described as follows. Where x is its East-West coordinate and y is its North-South coordinate, let z be its up-down coordinate. The height of the surface (measured in inches above sea-level) at the point with coordinates x and y is $z = xy^2 + x^3y$.

A) What is the gradient of this surface where $x = 4$ and $y = -2$?

B) Suppose that a fly traveling along this surface is at the point that has x, y coordinates $(x, y) = (4, -2)$ and is traveling in the direction $(1/\sqrt{10}, 3/\sqrt{10})$

in these coordinates. What is the vertical slope of the surface in the direction the fly is traveling? Is it going up or down?

C) If the fly is situated at the point with (x, y) coordinates $(4, -2)$, in what direction should it move to climb most steeply along the surface? (Describe this direction as a vector of length 1.)

7) Consider a three-dimensional surface described as follows. Where x is its East-West coordinate and y is its North-South coordinate, let z be its up-down coordinate. The height of the surface (measured in inches above sea-level) at the point with coordinates x and y is $z = xy^2 + x^3y$.

A) What is the gradient of this surface where $x = 4$ and $y = -2$?

B) Suppose that a fly traveling along this surface is at the point that has x, y coordinates $(x, y) = (4, -2)$ and is traveling in the direction $(1/\sqrt{10}, 3/\sqrt{10})$ in these coordinates. What is the vertical slope of the surface in the direction the fly is traveling? Is it going up or down?

C) If the fly is situated at the point with (x, y) coordinates $(4, -2)$, in what direction should it move to climb most steeply along the surface? (Describe this direction as a vector of length 1.)

8) We have a data set consisting 100 observations of commodity vectors x_j and price vectors p_j such that a certain consumer bought x_j when the price vector was p_j. We define the relations $\succeq_R$ and $\succ_R$ over commodity bundles such that $x_j \succeq_R x_k$ if $p_j x_j \geq p_j x_k$ and $x_j \succ_R x_k$ if $p_j x_j > p_k x_k$.

Suppose that this consumer has preferences that are locally non-satiated and transitive but not necessarily convex. From any budget the consumer chooses one of the best bundles that he can afford. Must it be true that if

$x_i \succeq_R x_j$ and $x_j \succeq_R x_k$, then NOT $x_k \succ x_i$. If so, prove it. If not, show a counterexample.