Adding together and using the budget constraint yields

$$\lambda = -a - b.$$

Substitute back into the first-order conditions to find

$$p_1 = \frac{a}{(a+b)x_1}$$
$$p_2 = \frac{b}{(a+b)x_2}.$$

These are the choices of (p_1, p_2) that minimize indirect utility. Now substitute these choices into the indirect utility function:

$$u(x_1, x_2) = -a \ln \frac{a}{(a+b)x_1} - b \ln \frac{b}{(a+b)x_2}$$
$$= a \ln x_1 + b \ln x_2 + \text{constant}.$$

This is the familiar Cobb-Douglas utility function.

8.7 Revealed preference

In our study of consumer behavior we have taken preferences as the primitive concept and derived the restrictions that the utility maximization model imposes on the observed demand functions. These restrictions are basically the Slutsky restrictions that the matrix of substitution terms be symmetric and negative semidefinite.

These restrictions are in principle observable, but in practice they leave something to be desired. After all, who has really seen a demand function? The best that we may hope for in practice is a list of the choices made under different circumstances. For example, we may have some observations on consumer behavior that take the form of a list of prices, $\mathbf{p}^t$, and the associated chosen consumption bundles, $\mathbf{x}^t$ for $t = 1, \ldots, T$. How can we tell whether these data could have been generated by a utility-maximizing consumer?

We will say that a utility function **rationalizes** the observed behavior $(\mathbf{p}^t, \mathbf{x}^t)$ for $t = 1, \cdots, T$ if $u(\mathbf{x}^t) \geq u(\mathbf{x})$ for all $\mathbf{x}$ such that $\mathbf{p}^t\mathbf{x}^t \geq \mathbf{p}^t\mathbf{x}$. That is, $u(\mathbf{x})$ rationalizes the observed behavior if it achieves its maximum value on the budget set at the chosen bundles. Suppose that the data were generated by such a maximization process. What observable restrictions must the observed choices satisfy?

Without any assumptions about $u(\mathbf{x})$ there is a trivial answer to this question, namely, no restrictions. For suppose that $u(\mathbf{x})$ were a constant function, so that the consumer was indifferent to all observed consumption bundles. Then there would be no restrictions imposed on the patters of observed choices: anything is possible.

To make the problem interesting, we have to rule out this trivial case. The easiest way to do this is to require the underlying utility function to be locally nonsatiated. Our question now becomes: what are the observable restrictions imposed by the maximization of a locally nonsatiated utility function?

First, we note that if $\mathbf{p}^t\mathbf{x}^t \geq \mathbf{p}^t\mathbf{x}$, then it must be the case that $u(\mathbf{x}^t) \geq u(\mathbf{x})$. Since $\mathbf{x}^t$ was chosen when $\mathbf{x}$ could have been chosen, the utility of $\mathbf{x}^t$ must be at least as large as the utility of $\mathbf{x}$. In this case we will say that $\mathbf{x}^t$ is **directly revealed preferred** to $\mathbf{x}$, and write $\mathbf{x}^t R^D \mathbf{x}$. As a consequence of this definition and the assumption that the data were generated by utility maximization, we can conclude that "$\mathbf{x}^t R^D \mathbf{x}$ implies $u(\mathbf{x}^t) \geq u(\mathbf{x})$."

Suppose that $\mathbf{p}^t\mathbf{x}^t > \mathbf{p}^t\mathbf{x}$. Does it follow that $u(\mathbf{x}^t) > u(\mathbf{x})$? It is not hard to show that local nonsatiation implies this conclusion. For we know from the previous paragraph that $u(\mathbf{x}^t) \geq u(\mathbf{x})$; if $u(\mathbf{x}^t) = u(\mathbf{x})$, then by local nonsatiation there would exist some other $\mathbf{x}'$ close enough to $\mathbf{x}$ so that $\mathbf{p}^t\mathbf{x}^t > \mathbf{p}^t\mathbf{x}'$ and $u(\mathbf{x}') > u(\mathbf{x}) = u(\mathbf{x}^t)$. This contradicts that hypothesis of utility maximization.

If $\mathbf{p}^t\mathbf{x}^t > \mathbf{p}^t\mathbf{x}$, we will say that $\mathbf{x}^t$ is **strictly directly revealed preferred** to $\mathbf{x}$ and write $\mathbf{x}^t P^D \mathbf{x}$.

Now suppose that we have a sequence of such revealed preference comparisons such that $\mathbf{x}^t R^D \mathbf{x}^j$, $\mathbf{x}^j R^D \mathbf{x}^k, \ldots, \mathbf{x}^n R^D \mathbf{x}$. In this case we will say that $\mathbf{x}^t$ is **revealed preferred** to $\mathbf{x}$ and write $\mathbf{x}^t R \mathbf{x}$. The relation R is sometimes called the **transitive closure** of the relation R^D. If we assume that the data were generated by utility maximization, it follows that "$\mathbf{x}^t R \mathbf{x}$ implies $u(\mathbf{x}^t) \geq u(\mathbf{x})$."

Consider two observations $\mathbf{x}^t$ and $\mathbf{x}^s$. We now have a way to determine whether $u(\mathbf{x}^t) \geq u(\mathbf{x}^s)$ and an observable condition to determine whether $u(\mathbf{x}^s) > u(\mathbf{x}^t)$. Obviously, these two conditions should not both be satisfied. This condition can be stated as the

GENERALIZED AXIOM OF REVEALED PREFERENCE. *If $\mathbf{x}^t$ is revealed preferred to $\mathbf{x}^s$, then $\mathbf{x}^s$ cannot be strictly directly revealed preferred to $\mathbf{x}^t$.*

Using the symbols defined above, we can also write this axiom as

GARP. *$\mathbf{x}^t\ R\ \mathbf{x}^s$ implies not $\mathbf{x}^s\ P^D\ \mathbf{x}^t$. In other words, $\mathbf{x}^t\ R\ \mathbf{x}^s$ implies $\mathbf{p}^s\mathbf{x}^s \leq \mathbf{p}^s\mathbf{x}^t$.*

As the name implies, GARP is a generalization of various other revealed preference tests. Here are two standard conditions.

WEAK AXIOM OF REVEALED PREFERENCE (WARP). *If* $\mathbf{x}^t\ R^D\ \mathbf{x}^s$ *and* $\mathbf{x}^t$ *is not equal to* $\mathbf{x}^s$, *then it is not the case that* $\mathbf{x}^s\ R^D\ \mathbf{x}^t$.

STRONG AXIOM OF REVEALED PREFERENCE (SARP). *If* $\mathbf{x}^t\ R\ \mathbf{x}^s$ *and* $\mathbf{x}^t$ *is not equal to* $\mathbf{x}^s$, *then it is not the case that* $\mathbf{x}^s\ R\ \mathbf{x}^t$.

Each of these axioms requires that there be a *unique* demand bundle at each budget, while GARP allows for multiple demanded bundles. Thus, GARP allows for flat spots in the indifference curves that generated the observed choices.

8.8 Sufficient conditions for maximization

If the data $(\mathbf{p}^t, \mathbf{x}^t)$ were generated by a utility-maximizing consumer with nonsatiated preferences, the data must satisfy GARP. Hence, GARP is an observable consequence of utility maximization. But does it express all the implications of that model? If some data satisfy this axiom, is it necessarily true that it must come from utility maximization, or at least be thought of in that way? Is GARP a *sufficient* condition for utility maximization?

It turns out that it is. If a finite set of data is consistent with GARP, then there exists a utility function that rationalizes the observed behavior—i.e., there exists a utility function that could have generated that behavior. Hence, GARP exhausts the list of restrictions imposed by the maximization model.

The following theorem is the nicest way to state this result.

Afriat's theorem. *Let* $(\mathbf{p}^t, \mathbf{x}^t)$ *for* $t = 1, \ldots, T$ *be a finite number of observations of price vectors and consumption bundles. Then the following conditions are equivalent.*

(1) There exists a locally nonsatiated utility function that rationalizes the data;

(2) The data satisfy GARP;

(3) There exist positive numbers (u^t, λ^t) *for* $t = 1, \ldots, T$ *that satisfy the Afriat inequalities:*

$$u^s \leq u^t + \lambda^t \mathbf{p}^t(\mathbf{x}^s - \mathbf{x}^t) \quad \textit{for all } t, s;$$

(4) There exists a locally nonsatiated, continuous, concave, monotonic utility function that rationalizes the data.

Proof. We have already seen that (1) implies (2). The proof that (2) implies (3) is omitted; see Varian (1982a) for the argument. The proof that (4) implies (1) is trivial. All that is left is the proof that (3) implies (4).

We establish this implication constructively by exhibiting a utility function that does the trick. Define

$$u(\mathbf{x}) = \min_t \{u^t + \lambda^t \mathbf{p}^t(\mathbf{x} - \mathbf{x}^t)\}.$$

Note that this function is continuous. As long as $\mathbf{p}^t \geq \mathbf{0}$ and no $\mathbf{p}^t = \mathbf{0}$, the function will be locally nonsatiated and monotonic. It is also not difficult to show that it is concave. Geometrically, this function is just the lower envelope of a finite number of hyperplanes.

We need to show that this function rationalizes the data; that is, when prices are $\mathbf{p}^t$, this utility function achieves its constrained maximum at $\mathbf{x}^t$. First we show that $u(\mathbf{x}^t) = u^t$. If this were not the case, we would have

$$u(\mathbf{x}^t) = u^m + \lambda^m \mathbf{p}^m(\mathbf{x}^t - \mathbf{x}^m) < u^t.$$

But this violates one of the Afriat inequalities. Hence, $u(\mathbf{x}^t) = u^t$.

Now suppose that $\mathbf{p}^s\mathbf{x}^s \geq \mathbf{p}^s\mathbf{x}$. It follows that

$$u(\mathbf{x}) = \min_t \{u^t + \lambda^t \mathbf{p}^t(\mathbf{x} - \mathbf{x}^t)\} \leq u^s + \lambda^s \mathbf{p}^s(\mathbf{x} - \mathbf{x}^s) \leq u^s = u(\mathbf{x}^s).$$

This shows that $u(\mathbf{x}^s) \geq u(\mathbf{x})$ for all $\mathbf{x}$ such that $\mathbf{p}^s\mathbf{x} \leq \mathbf{p}^s\mathbf{x}^s$. In other words, $u(\mathbf{x})$ rationalizes the observed choices. ∎

The utility function defined in the proof of Afriat's theorem has a natural interpretation. Suppose that $u(\mathbf{x})$ is a concave, differentiable utility function that rationalizes the observed choices. The fact that $u(\mathbf{x})$ is differentiable implies it must satisfy the T first-order conditions

$$\mathbf{D}u(\mathbf{x}^t) = \lambda^t \mathbf{p}^t. \tag{8.3}$$

The fact that $u(\mathbf{x})$ is concave implies that it must satisfy the concavity conditions

$$u(\mathbf{x}^t) \leq u(\mathbf{x}^s) + \mathbf{D}u(\mathbf{x}^s)(\mathbf{x}^t - \mathbf{x}^s). \tag{8.4}$$

Substituting from (8.3) into (8.4), we have

$$u(\mathbf{x}^t) \leq u(\mathbf{x}^s) + \lambda^s \mathbf{p}^s(\mathbf{x}^t - \mathbf{x}^s).$$

Hence, the Afriat numbers u^t and λ^t can be interpreted as utility levels and marginal utilities that are consistent with the observed choices.

The most remarkable implication of Afriat's theorem is that (1) implies (4): if there is any locally nonsatiated utility function at all that rationalizes the data, there must exist a continuous, monotonic, and concave utility function that rationalizes the data. This is similar to the observation made in Chapter 6, page 83, where we showed that if there were nonconvex parts of the input requirement set, no cost minimizer would ever choose to operate there.

The same is true for utility maximization. If the underlying utility function had the "wrong" curvature at some points, we would never observe choices being made at such points because they wouldn't satisfy the right second-order conditions. Hence market data do not allow us to reject the hypotheses of convexity and monotonicity of preferences.

8.9 Comparative statics using revealed preference

Since GARP is a necessary and sufficient condition for utility maximization, it must imply conditions analogous to comparative statics results derived earlier. These include the Slutsky decomposition of a price change into the income and the substitution effects and the fact that the own substitution effect is negative.

Let us begin with the latter result. When we consider finite changes in a price rather than just infinitesimal changes, there are two possible definitions of the compensated demand. The first definition is the natural extension of our earlier definition—namely, the demand for the good in question if we change the level of income so as to restore the original level of utility. That is, the value of the compensated demand for good i when prices change from $\mathbf{p}$ to $\mathbf{p} + \Delta\mathbf{p}$ is just $x_i(\mathbf{p} + \Delta\mathbf{p}, m + \Delta m) \equiv x_i(\mathbf{p} + \Delta\mathbf{p}, e(\mathbf{p}+\Delta\mathbf{p}, u))$, where u is the original level of utility achieved at $(\mathbf{p}, m)$. This notion of compensation is known as the **Hicksian compensation**.

The second notion of compensated demand when prices change from $\mathbf{p}$ to $\mathbf{p} + \Delta\mathbf{p}$ is known as the **Slutsky compensation**. It is the level of demand that arises when income is changed so as to make the original level of *consumption* possible. This is easily described by the following equations. We want the change in income, Δm, necessary to allow for the old level of consumption, $\mathbf{x}(\mathbf{p}, m)$, to be feasible at the new prices, $\mathbf{p} + \Delta\mathbf{p}$. That is

$$(\mathbf{p} + \Delta\mathbf{p})\mathbf{x}(\mathbf{p}, m) = m + \Delta m.$$

Since $\mathbf{p}\ \mathbf{x}(\mathbf{p}, m) = m$, this reduces to $\Delta\mathbf{p}\ \mathbf{x}(\mathbf{p}, m) = \Delta m$.

The difference between the two notions of compensation is illustrated in Figure 8.6. The Slutsky notion is directly measurable without knowledge of the preferences, but Hicksian notion is more convenient for analytic work.

For infinitesimal changes in price there is no need to distinguish between the two concepts since they coincide. We can prove this simply by examining the expenditure function. If the price of good j changes by dp_j, we need to change expenditure by $(\partial e(\mathbf{p}, u)/\partial p_j)dp_j$ to keep utility constant.

If we want to keep the old level of consumption feasible, we need to change income by $x_j dp_j$. By the derivative property of the expenditure function, these two magnitudes are the same.

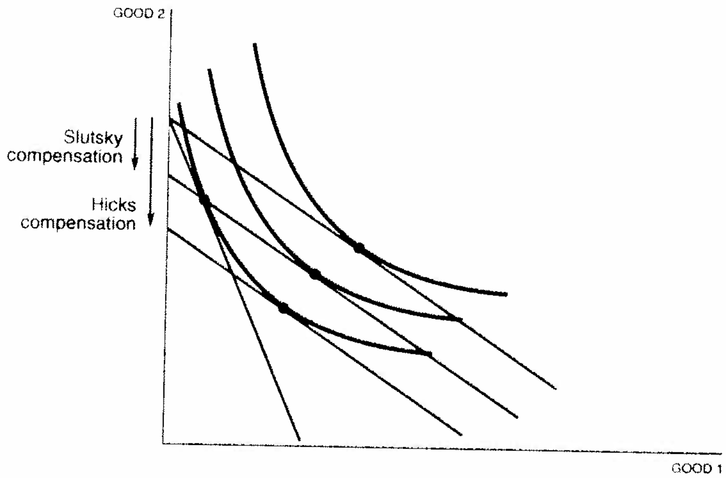


Figure 8.6 **Hicks and Slutsky compensation.** Hicks compensation is an amount of money that makes the original level of utility affordable. Slutsky compensation is an amount of money that makes the original consumption bundle achievable.

Whichever definition you prefer, we can still use revealed preference to prove that "the compensated own-price effect is negative." Suppose we consider the Hicksian definition. We start with a price vector $\mathbf{p}$ and let $\mathbf{x} = \mathbf{x}(\mathbf{p}, m)$ be the demanded bundle. The price vector changes to $\mathbf{p} + \boldsymbol{\Delta}\mathbf{p}$, and the compensated demand, therefore, changes to $\mathbf{x}(\mathbf{p} + \boldsymbol{\Delta}\mathbf{p}, m + \Delta m)$, where Δm is the amount necessary to make $\mathbf{x}(\mathbf{p} + \boldsymbol{\Delta}\mathbf{p}, m + \Delta m)$ indifferent to $\mathbf{x}(\mathbf{p}, m)$.

Since $\mathbf{x}(\mathbf{p}, m)$ and $\mathbf{x}(\mathbf{p} + \boldsymbol{\Delta}\mathbf{p}, m + \Delta m)$ are indifferent to each other, neither can be strictly directly revealed preferred to the other. That is, we must have

$$\mathbf{p}\,\mathbf{x}(\mathbf{p}, m) \leq \mathbf{p}\,\mathbf{x}\,(\mathbf{p} + \boldsymbol{\Delta}\mathbf{p}, m + \Delta m)$$
$$(\mathbf{p} + \boldsymbol{\Delta}\mathbf{p})\mathbf{x}(\mathbf{p} + \boldsymbol{\Delta}\mathbf{p}, m + \Delta m) \leq (\mathbf{p} + \boldsymbol{\Delta}\mathbf{p})\mathbf{x}(\mathbf{p}, m).$$

Adding these inequalities together, we have

$$\boldsymbol{\Delta}\mathbf{p}[\mathbf{x}(\mathbf{p} + \boldsymbol{\Delta}\mathbf{p}, m + \Delta m) - \mathbf{x}(\mathbf{p}, m)] \leq 0.$$

Letting $\Delta\mathbf{x} = \mathbf{x}(\mathbf{p} + \boldsymbol{\Delta}\mathbf{p}, m + \Delta m) - \mathbf{x}(\mathbf{p}, m)$, this becomes

$$\Delta\mathbf{p}\;\boldsymbol{\Delta}\mathbf{x} \leq 0.$$

Suppose that only one price has changed so that $\mathbf{\Delta p} = (0, \cdots, \Delta p_i, \cdots, 0)$. Then this inequality implies that x_i must change in the opposite direction.

We now turn to the Slutsky definition. We keep the same notation as before, but now interpret Δm as the change in income necessary to make the old consumption bundle affordable. Since $\mathbf{x}(\mathbf{p}, m)$ is thus by hypothesis a feasible level of consumption at $\mathbf{p} + \mathbf{\Delta p}$, the bundle actually chosen at $\mathbf{p} + \mathbf{\Delta p}$ cannot be revealed worse than $\mathbf{x}(\mathbf{p}, m)$. That is,

$$\mathbf{p}\ \mathbf{x}(\mathbf{p}, m) \leq \mathbf{p}\ \mathbf{x}(\mathbf{p} + \mathbf{\Delta p}, m + \Delta m).$$

Since $(\mathbf{p} + \mathbf{\Delta p})\mathbf{x}(\mathbf{p} + \mathbf{\Delta p}, m + \Delta m) = (\mathbf{p} + \mathbf{\Delta p})\mathbf{x}(\mathbf{p}, m)$ by construction of Δm, we can subtract this equality from the above inequality to find

$$\mathbf{\Delta p}\ \mathbf{\Delta x} \leq 0,$$

just as before.

8.10 The discrete version of the Slutsky equation

We turn now to the task of deriving the Slutsky equation. We derived this equation earlier by differentiating an identity involving Hicksian and Marshallian demands. We start by writing the following arithmetic identity:

$$x_i(\mathbf{p} + \mathbf{\Delta p}, m) - x_i(\mathbf{p}, m) = x_i(\mathbf{p} + \mathbf{\Delta p}, m + \Delta m) - x_i(\mathbf{p}, m) - [x_i(\mathbf{p} + \mathbf{\Delta p}, m + \Delta m) - x_i(\mathbf{p} + \mathbf{\Delta p}, m)].$$

Note that this is true by the ordinary rule of algebra.

Suppose that $\mathbf{\Delta p} = (0, \cdots, \Delta p_j, \cdots, 0)$. Then the compensating change in income—in the Slutsky sense—is $\Delta m = x_j(\mathbf{p}, m)\Delta p_j$. If we divide each side of the above identity by Δp_j and use the fact that $\Delta p_j = \Delta m / x_j(\mathbf{p}, m)$, we have

$$\frac{x_i(\mathbf{p} + \mathbf{\Delta p}, m) - x_i(\mathbf{p}, m)}{\Delta p_j} = \frac{x_i(\mathbf{p} + \mathbf{\Delta p}, m + \Delta m) - x_i(\mathbf{p}, m)}{\Delta p_j} - x_j(\mathbf{p}, m)\frac{[x_i(\mathbf{p} + \mathbf{\Delta p}, m + \Delta m) - x_i(\mathbf{p} + \mathbf{\Delta p}, m)]}{\Delta m}.$$

Interpreting each of the terms in this expression, we can write

$$\frac{\Delta x_i}{\Delta p_j} = \left.\frac{\Delta x_i}{\Delta p_j}\right|_{\text{comp}} - x_j \frac{\Delta x_i}{\Delta m}.$$

Note that this last equation is simply a discrete analog of the Slutsky equation. The term on the left-hand side is how the demand for good i changes as price j changes. This is decomposed into the substitution effect—how the demand for good i changes when price j changes and income is also changed so as to keep the original level of consumption possible—and the income effect—how the demand for good i changes when prices are held constant but income changes times the demand for good j. The Slutsky decomposition of a price change is illustrated in Figure 8.7.

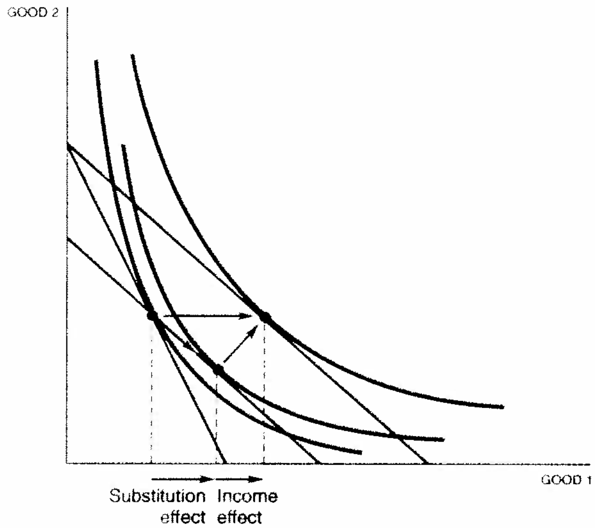


Figure 8.7 **Slutsky decomposition of a price change.** First pivot the budget line around the original consumption bundle and then shift it out to the final choice.

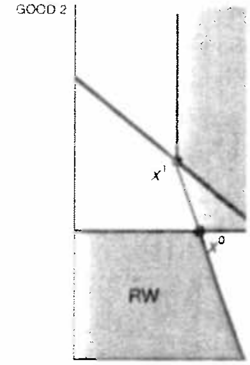
8.11 Recoverability

Since the revealed preference conditions are a complete set of the restrictions imposed by utility-maximizing behavior, they must contain all of the information available about the underlying preferences. It is more-or-less obvious how to use the revealed preference relations to determine the preferences among the *observed* choices, $\mathbf{x}^t$, for $t = 1, \ldots, T$. However, it is less obvious to use the revealed preference relations to tell you about preference relations between choices that have never been observed.

This is easiest to see using an example. Figure 8.8 depicts a a single observation of choice behavior, $(\mathbf{p}^1, \mathbf{x}^1)$. What does this choice imply about the indifference curve through a bundle $\mathbf{x}^0$? Note that $\mathbf{x}^0$ has not been previously observed; in particular, we have no data about the prices at which $\mathbf{x}^0$ would be an optimal choice.

Let's try to use revealed preference to "bound" the indifference curve through $\mathbf{x}^0$. First, we observe that $\mathbf{x}^1$ is revealed preferred to $\mathbf{x}^0$. Assume that preferences are convex and monotonic. Then all the bundles on the line segment connecting $\mathbf{x}^0$ and $\mathbf{x}^1$ must be at least as good as x^0, and all the bundles that lie to the northeast of this bundle are at least as good as $\mathbf{x}^0$. Call this set of bundles $RP(x^0)$, for "revealed preferred" to $\mathbf{x}^0$. It is not difficult to show that this is the best "inner bound" to the upper contour set through the point $\mathbf{x}^0$.

To derive the best outer bound, we must consider all possible budget lines



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Notes

The dual proof of th
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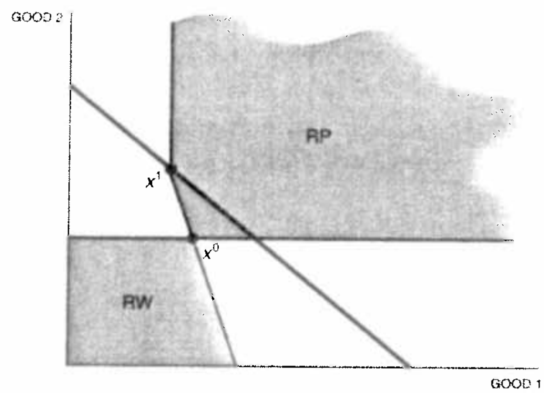

Figure 8.8 **Inner and outer bounds.** *RP* is the inner bound to the indifference curve through $\mathbf{x}^0$; the complement of *RW* is the outer bound.

passing through $\mathbf{x}^0$. Let *RW* be the set of all bundles that are revealed worse than $\mathbf{x}^0$ for *all* these budget lines. The bundles in *RW* are certain to be worse than $\mathbf{x}^0$ no matter what budget line is used.

The outer bound to the upper contour set at $\mathbf{x}^0$ is then defined to be the complement of this set: NRW = all bundles not in RW. This is the best outer bound in the sense that any bundle not in this set cannot ever be revealed preferred to $\mathbf{x}^0$ by a consistent utility-maximizing consumer. Why? Because by construction, a bundle that is not in $NRW(\mathbf{x}^0)$ must be in $RW(\mathbf{x}^0)$ in which case it would be revealed *worse* than $\mathbf{x}^0$.

In the case of a single observed choice, the bounds are not very tight. But with many choices, the bounds can become quite close together, effectively trapping the true indifference curve between them. See Figure 8.9 for an illustrative example. It is worth tracing through the construction of these bounds to make sure that you understand where they come from. Once we have constructed the inner and outer bounds for the upper contour sets, we have recovered essentially all the information about preferences that is contained in the observed demand behavior. Hence, the construction of *RP* and *RW* is analogous to solving the integrability equations.

Our construction of *RP* and *RW* up until this point has been graphical. However, it is possible to generalize this analysis to multiple goods. It turns out that determining whether one bundle is revealed preferred or revealed worse than another involves checking to see whether a solution exists to a particular set of linear inequalities.

Notes

The dual proof of the Slutsky equation given here follows McKenzie (1957) and Cook (1972). A detailed treatment of integrability may be found

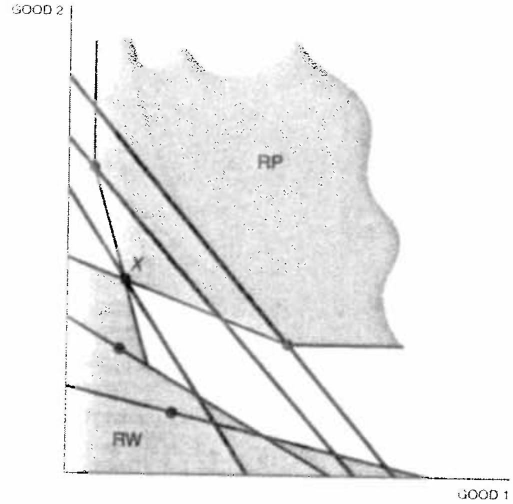

Figure 8.9 **Inner and outer bounds.** When there are several observations, the inner and outer bounds can be quite tight.

in Hurwicz & Uzawa (1971). The idea of revealed preferences is due to Samuelson (1948). The approach taken here follows that of Afriat (1967) and Varian (1982a). The derivation of the Slutsky equation using revealed preference follows Yokoyama (1968).

Exercises

8.1. Frank Fisher's expenditure function is $e(\mathbf{p}, u)$. His demand function for jokes is $x_j(\mathbf{p}, m)$, where $\mathbf{p}$ is vector of prices and $m \gg 0$ is his income. Show that jokes are a normal good for Frank if and only if $\partial^2 e/\partial p_j \partial u > 0$.

8.2. Calculate the substitution matrix for the Cobb-Douglas demand system with two goods. Verify that the diagonal terms are negative and the cross-price effects are symmetric.

8.3. Suppose that a consumer has a linear demand function $x = ap + bm + c$. Write down the differential equation you would need to solve to find the money metric utility function. If you can, solve this differential equation.

8.4. Suppose that a consumer has a semi-log demand function $\ln x = ap + bm + c$. Write down the differential equation you would need to solve to find the money metric utility function. If you can, solve this differential equation.

8.5. Find the demar
$u(x_1, x_2) = x_1^{\frac{3}{2}} x_2$ an

8.6. Use the utility 1
$m = p_1 x_1 + p_2 x_2$ to

8.7. Extend the pre
$\alpha_1)^{\beta_1}(x_2 - \alpha_2)^{\beta_2}$ an
terms $\left(\frac{\partial h_j(\mathbf{p}, u)}{\partial p_i}\right)$.

8.8. Repeat the prev
show that all the pre

8.9. Preferences are r
direct utility functior
are now represented
$\psi(\cdot)$, show that $e(\mathbf{p}, u$
and $\mathbf{h}(\mathbf{p}, u)$ by $\mathbf{h}(\mathbf{p},$
$\mathbf{x}(\mathbf{p}, m)$ are unaffecte

8.10. Consider a two
where x_1 represents l
second period's consu
consume in each peri
future consumption a

where p_1 and p_2 are t

(a) Derive the Slu
income depends on th
prices: $m = p_1\bar{x}_1 + p_2$

(b) Assume that D
down, will Dave be be

(c) What is the rat

8.11. Consider a cons
price of the goods are
he demands (2, 1). No
maximizing utility?