Notes on Revealed Preference

Ted Bergstrom, UCSB Economics 210A

November 17, 2016

We have shown how to derive demand functions, utility functions, and choice behavior, starting from some fundamental assumptions about preferences. Let us now work the other way around. Let us make some postulates about observable demand behavior and make assumptions about choices that imply "rational" preferences.

Suppose that all we observe are choices made by a price-taking consumer with some alternative budgets. Let us maintain the assumption that preferences are locally nonsatiated. (Reminder–local nonsatiation means you will spend your entire budget.

The Weak Axiom of Revealed Preference

Suppose that the consumer chooses bundle x^0 at price vector p^0 and bundle bundle x^1 at price vector p^1, where $x^0 \neq x^1$. If $p^0x^1 \leq p^0x^0$, we say that x^0 *is directly revealed preferred to* x^1. Let us write this as $x^0 \succeq_{DRP} x_1$. Draw picture. You choose x^0 when you could have had x^1.

WARP is the assumption that if x^0 is directly revealed preferred to x^1, then it can not be that x^1 is directly revealed preferred to x^0. Another way of saying this is to say that $\succeq_{DRP}$ is an *asymmetric* relation. Equivalently, if bundle x^0 is purchased at price vector p^0 and if x^1 is purchased at price vector p^1, then if $p^0x^1 \leq p^0x^0$, it must be that $p^1x^0 > p^1x^1$. In other words, if you buy x^0 when you could have had x^1, then if you buy x^1, you can't afford x^0.

Let us define the relation $\succeq_{RP}$ (revealed preferred) saying $x^0 \succeq_{RP} x^n$ to be the "transitive closure" of $\succeq_{DRP}$. This means means that there is some chain of x^i's such that $x^i \succeq x_{i+1}$ for $i = 0$ to $i = n-1$. Draw picture to show what this means. The strong axiom of revealed preference SARP says that if $x^0 \succeq_{RP} x^1$, then not $x^1 \succeq_{RP} x^0$. In other words, the relation $\succeq_{DRP}$ has no cycles.

Historically, this is the way that WARP and SARP were first stated and is still in common use. But it is kind of awkward that it does not deal appropriately with the possibility of "ties" for best affordable bundle. A rational consumer might be indifferent between two bundles the maximize his preferences on some budget. Such a consumer would violate WARP. This could happen if preferences are not strictly convex. We would like a revealed preference theory that doesn't require strictly convex preferences. Fortunately there is such a theory.

The following result is not surprising, but to understand revealed preference theory, it is important to state it and see why it is true.

Lemma 1. *Suppose that a consumer has a continuous utility function* $u(x)$*, and for any price vector* $p >> 0$ *and income* $m > 0$*, this consumer chooses* $x(p)$ *to maximize* $u(\cdot)$ *subject to the constraint that* $px(p) \leq m$*. Then if* $x \succeq_{RP} y$*, it must be that* $u(x) \geq u(y)$*.*

Proof. First we show that if $x \succeq_{DRP} y$, then $u(x) \geq u(y)$. Suppose that $x \succeq_{DRP} y$, then for some price vector p, $py \leq px$, and x is chosen when income is px, This implies that $u(x) \geq u(x')$ for all x' such that $px' \leq px$. Therefore $u(x) \geq u(y)$.

Now $x \succeq_{RP} y$, Then there must be a sequence of bundles $x^1, \ldots, x^n$, such that $x \succeq_{DRP} x_1$, $x^n \succeq_{DRP} y$, and $x^i \succeq_{DRP} x^{i+1}$, for $i = 1 \ldots N-1$. But this implies that $u(x) \geq u(x^1) \geq u(x^n) \geq u(y)$. So by transitiveity of the ordering $\geq$ of the real numbers, it must be that $u(x) \geq u(y)$.

□

GARP

Let us define x to be strictly directly revealed preferred to y by a consumer if that consumer chooses x when the price vector is p and at these prices can "more than afford" y. . That is if $px > py$. If a consumer is locally nonsatiated and has transitive preferences, then this inequality would imply that she chooses x when she could afford some z that she likes better than y. So she must like x at least as well as z and z better than y and hence y better than z. Let us denote the relation x is strictly directly preferred to y by $x \succ_{DRP} y$.

With this addition, we can state the following axiom, known as GARP (Generalized Axiom of Revealed Preference) "If x is revealed preferred to y, (directly or indirectly), then it can not be that y is strictly directly preferred to x."

It is clear that an optimizing consumer with complete transitive preferences will never violate GARP. (Show this.) A more subtle result is that any finite set of data, showing combinations of prices and bundles chosen at these prices by a single consumer can be explained as maximizing behavior by an agent with transitive complete preferences if and only if the data satisfies GARP.

The single step version of GARP (which corresponds to WARP, but allows for ties) is "If x is directly revealed preferred to y, then it can not be that y is strictly directly preferred to x." It turns out that this is not sufficient to imply GARP and hence not sufficient to imply that the data can be explained as maximizing behavior of a rational agent. You will find an example where WARP is satisfied but GARP is not in Workouts Problem 7.6,