

Three Utility Problems

Problem 1 is found in David Kreps' Microeconomic Foundations I.

Problem 2 is a variation of a problem from Kreps.

Problem 1: Consider the following two utility functions defined on \mathfrak{R}_+^2 .

$$U_1(x_1, x_2) = \begin{cases} x_1 x_2 & \text{if } x_1 x_2 < 4 \\ 4 & \text{if } 4 \leq x_1 x_2 \leq 8 \\ x_1 x_2 - 4 & \text{if } 8 \leq x_1 x_2 \end{cases}$$

$$U_2(x_1, x_2) = \begin{cases} x_1 x_2 & \text{if } x_1 x_2 < 4 \\ 4 & \text{if } x_1 x_2 = 4 \text{ and } x_1 \geq x_2 \\ 5 & \text{if } x_1 x_2 = 4 \text{ and } x_1 < x_2 \\ x_1 x_2 + 1 & \text{if } x_1 x_2 > 4 \end{cases}$$

a) Show that both of these two utility functions represent convex preferences.

b) Are the preferences represented by either or both of these functions semi-strictly convex?

c) Are the preferences represented by either or both of these functions continuous?

Problem 2

Consider the following utility function defined on the domain $X = \{(x_1, x_2) | 0 \leq x_1, 0 \leq x_2 < 2\}$

$$U(x_1, x_2) = \begin{cases} \frac{2(1+x_2)}{2-x_1} - 1 & \text{if } x_1 + x_2 \neq 1 \\ x_1 + x_2 & \text{if } x_1 + x_2 = 1 \end{cases}$$

a) Draw some indifference curves for an individual with this utility function.

b) Does this utility function represents preferences that are monotone increasing? continuous? convex?

c) Find the Marshallian demand function for a consumer with this utility function.

Problem 3 (Through the looking Glass?)

Suppose that the utility function is as in Problem 2, but that its domain is $X = \{(x_1, x_2) | 0 \leq x_1, 0 \leq x_2, \text{ and } x_2 \neq 2\}$ Draw the indifference map for this person.