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Final Examination: Economics 210A

December, 2015

1) The island nation of Santa Felicidad has N skilled workers and N unskilled workers. A skilled worker can earn $\$w_S$ per day if she works all the time and an unskilled worker can earn $\$w_U$ per day if he works all the time. A worker who can earn w per day and spends the fraction ℓ of his or her time NOT working will earn $(1-\ell)w$ per day. There is a single consumption good. Preferences of all workers are represented by the utility function

$$U(c,\ell) = \ln c + \ln \ell \tag{1}$$

where c is the amount of the consumption good consumed per day and ℓ is the fraction of the consumer's time that is spent not working. Suppose that the government is able to recognize who is skilled and who is not skilled and collects a lump sum tax of t from each skilled worker and pays a lump sum subsidy of t to each unskilled worker.

A) Find the fraction of time that each type of worker will spend at leisure as a function of that type of worker's wage and the variable t.

A skilled worker will take the leisure for the fraction

$$\ell_S = \frac{w_S - t}{2w_S} = \frac{1}{2} - \frac{t}{2w_S}$$

and an unskilled worker will take leisure for the fraction

$$\ell_U = \frac{w_U + t}{2w_U} = \frac{1}{2} + \frac{t}{2w_U}.$$

B) A politician who opposes taxing the rich says that "Redistributive taxes harm the economy. An increase in such taxes will reduce the total supply of labor." A politician who favors taxing the rich says that "Tax increases have no effect on the total value of labor earnings and will actually increase the "average productivity of labor". To evaluate the claims of these politicians, do the following for the economy described in this problem: (i) Show how the total number of hours worked by San Felicidad workers of each type changes as t changes. (Remember that skilled workers pay a tax of t and unskilled workers get a subsidy of t.) As t increases, does the total number of hours worked by all workers increase, decrease or stay constant? (ii) Show how the total labor earnings of each type of worker changes as t changes. As t increases, does the total labor earnings of all laborers in the country increase, decrease or stay constant? (iii) Determine whether an increase in t increases, decreases, or leaves unchanged, the "average productivity of labor" as measured by the ratio of total labor earnings to total hours worked by San Felicidad workers.

Total hours of labor is proportional to

$$1-\ell_S+1-\ell_U=1+t\left(\frac{1}{2w_S}-\frac{1}{2w_U}\right).$$

The derivative of this expression with respect to t is

$$\frac{1}{2w_S}-\frac{1}{2w_U}<0.$$

So the total number of hours of work decreases as the tax rate increases. Total labor earnings are equal to

$$w_S\left(\frac{1}{2}+\frac{t}{w_S}\right)+w_U\left(\frac{1}{2}-\frac{t}{w_U}\right)=\frac{w_S}{2}+\frac{w_U}{2}.$$

Thus total earnings are constant as t changes.

C) Find the indirect utility function $v(w,\tau)$ where $v(w,\tau)$ is the utility of a worker with utility function given by Equation 1 who is paid a wage of w per day and who pays a tax of τ to the government, where τ could be either positive or negative and where $\tau>0$ means a tax and $\tau<0$ means a subsidy.

$$v(w,\tau)=\ln\frac{w+\tau}{2w}+\ln\frac{w+\tau}{2}=2\ln\left(w+\tau\right)-\ln w-2\ln 2$$

D) Suppose that young workers don't know whether they will be skilled or unskilled and each of them thinks the probability of either outcome is 1/2. Suppose that they are expected utility maximizers with von Neumann-Morgenstern utility functions given by Equation 1. What tax rate t^* paid by the skilled and collected by the unskilled would maximize expected utility for these young workers. Explain.

If the tax rate is t, the expected utility of a young worker is

$$\frac{1}{2}\left(2\ln\left(w_S-t\right)-\ln w_S-2\ln 2\right)+\frac{1}{2}\left(2\ln\left(w_U+t\right)-\ln w_U-2\ln 2\right)$$

The derivative of this expected utility with respect to t is equal to zero when

$$-\frac{1}{w_S-t}=\frac{1}{w_U+t}$$

This implies that $w_S-t=w_U+t$ which implies that $t^*=\frac{w_S-w_t}{2}$

2 A consumer has utility function

$$U(x_1, x_2) = \left(x_1^{1/2} + x_2^{1/2}\right)^2. \tag{2}$$

A) What is this consumer's elasticity of substitution between the two goods? Find this consumer's Marshallian demand function for each of the two goods. Then find a simple expression for the fraction of her income that she spends on each good.

The elasticity of substitution is 1/2. The Marshallian demand functions are

$$x_1(p, m) = \frac{p_2}{p_1} \frac{m}{p_1 + p_2}$$

$$x_2(p, m) = \frac{p_1}{p_2} \frac{m}{p_1 + p_2}.$$

The consumer spends the fraction $\frac{p_2}{p_1+p_2}$ *of income on good 1 and the fraction* $\frac{p_2}{p_1+p_2}$ *on good 2.*

B) Find this consumer's indirect utility function.

The indirect utility function can be written as

$$m\left(\frac{1}{p_1} + \frac{1}{p_2}\right)$$

You may have an equivalent answer in a less simple form.

3 Consider a pure exchange economy with 200 consumers. All of them have the utility function given in Equation 2 of Question 1. There are 100 Type A consumers who have initial endowment of 90 units of good 1 and no units of good 2. There are 100 Type B consumers who have initial endowment of 10 units of good 2 and no units of good 1. Let Good 1 be the numeraire. Find the competitive equilibrium price of good 2 for this economy. Find the competitive equilibrium consumption of each good for each type of consumer. (Hint: Finding the competitive equilibrium price requires only a very simple calculation.)

All consumers have homothetic and identical preferences. Since all face the same price ratio, all must consume the goods in the same ratio. Since supply equals demand this ratio must be the ratio of the total supply of the two goods, which is 9 units of good 1 per unit of good 2. In equilibrium it must be that

$$\frac{p_2}{p_1} = \left(\frac{90 \times 100}{10 \times 1000}\right)^{1/2} = 3.$$

When good 1 is the numeraire, the competive price of good 2 is 3. In competitive equilibrium, the income of each type 1 consumer is 90 and the income of each type 2 consumer 30.

From the Marshallian demand functions we see that at these prices the type 1 consumers consume $\frac{3}{4} \times 90 = 67.5$ *units of good 1 and the type 2 consumers consume* $\frac{3}{4} \times 30 = 22.5$ *units of good 1. Type 1 consumers consume* $\frac{1}{3}\frac{90}{4} = 7.5$ *units of good 2. Type 2 consumers consume 2.5 units of good 2.*

4 A) If a decision-maker has constant absolute risk aversion, what functional form must that person's von Neumann Morgenstern utility function take? Show that your answer is correct.

The utility function must be of the form

$$u(x) = a - be^{-cx}$$

In this case the coefficient of absolute risk aversion is

$$-u''(x)/u'(x) = \frac{bc^2e^{-cx}}{-bce^{-cx}} = c.$$

B) Suppose that a decision-maker has constant absolute risk aversion and initial wealth w and prefers not to take between a gamble such that with probability 1/2 he would win \$$x$ and with probability 1/2 he would lose \$$y$. Is it possible that if this individual's wealth were much greater, he would want to accept the gamble? Prove your answer.

No. A decision-maker with CARA and wealth w will not accept this gamble if

$$\frac{1}{2}e^{-c(w+x)} + \frac{1}{2}e^{-c(w-y)} = e^{cw}\left(\frac{1}{2}e^{cx} + \frac{1}{2}e^{-cy}\right) < 0.$$

But this is equivalent to

$$\frac{1}{2}e^{cx} + \frac{1}{2}e^{-cy} < 0,$$

which in turn implies that for all w',

$$e^{cw'}\left(\frac{1}{2}e^{cx} + \frac{1}{2}e^{-cy}\right) < 0.$$

This implies that no matter what his wealth is, the decision-maker will not take this bet.

5 State Walras' Law. Provide some assumptions under which Walras' Law holds and prove that Walras' Law holds under these assumptions.

6 A) State (i) the Weak axiom of revealed preference, (ii) the Strong axiom of revealed preference and (iii) the generalized axiom of revealed preference. *(Be sure to define "is revealed preferred to.")*

B) Show that a consumer who makes choices thar maximize a continuous, transitive, strictly convex preference relation must satisfy the strong axiom of revealed preference.

7) Consider a pure exchange economy with two consumers and two goods. Consumer 1 has an initial endowment of ω^x units of good x and 0 units of good y. Consumer 2 has an initial endowment of ω^y units of good y and 0 units of good x. Consumer 1's utility function is $U(x_1, y_1) = \min\{x_1, y_1\}$ and consumer 2's utility function is $U(x_1, y_1) = x_1 y_1$, where x_i and y_i are consumptions of goods x and y respectively by consumer i.

A) Draw an Edgeworth box representation of this economy.

B) Suppose that $y_0 < 2x_0$. Where good 1 is the numeraire, find the competitive equilibrium price for good 2. Also find the competitive equilibrium quantities consumed by each consumer in competitive equilibrium.

At price p for good 2, Consumer 1 demands equal amounts of goods 1 and 2. From the budget equation this implies that consumer 1's demand for good y *is* $y_1 = \frac{\omega^x}{1+p}$. *Consumer 2's demand for good* y *is* $\omega^y/2$. *Then excess demand for* y *is zero if*

$$\frac{\omega^x}{1+p} + \frac{\omega^y}{2} = \omega^y.$$

Therefore excess demand is zero when $p = 2\frac{\omega^x}{\omega^y} - 1$.

For Extra credit–only if you have extra time. What can you say about competitive equilibrium for this economy if $y_0 > 2x_0$.