Final Exam, Economics 210A, December 2013

There are 7 questions. Answer as many as you can... Good luck!

1) Find the indirect utility function and the expenditure function for a consumer with each of the following utility functions.

i) $U(x_1, x_2) = \min\{x_1, x_2\}$

$$v(p, m) = \frac{m}{p_1 + p_2} \text{ and } \; e(p, u) = u(p_1 + p_2)$$

ii) $U(x_1, x_2) = \sqrt{x_1 x_2}$

$$v(p, m) = \frac{m}{2\sqrt{p_1 p_1}} \text{ and } e(p, u) = 2u\sqrt{p_1 p_2}$$

iii) $U(x_1, x_2) = x_1 + x_2$

$$v(p, m) = \frac{m}{\min\{p_1, p_2\}} \text{ and } e(p, u) = \min\{p_1, p_2\}$$

iv) $U(x_1, x_2) = \max\{x_1, x_2\}$

$$v(p, m) = \frac{m}{\min\{p_1, p_2\}} \text{ and } e(p, u) = \min\{p_1, p_2\}$$

2) Find the Marshallian demand function for a consumer with each of the following utility functions: (Hint: You may want to think about making use of a two-stage process, using indirect utility.)

i) $U(x_1, x_2, x_3, x_4) = \min\{\sqrt{x_1 x_2}, \sqrt{x_3 x_4}\}$

Suppose that he has total income M and spends m on goods 1 and 2 and m' on goods 3 and 4. Then he will choose x_1 and x_2 to maximize $\sqrt{x_1 x_2}$ subject to $p_1 x_1 + p_2 x_2 = m$ and he will choose x_3 and x_4 to maximize $\sqrt{x_3 x_4}$ subject to $p_3 x_3 + p_4 x_4 = m'$. Knowing the indirect utility functions, we can see that when he does this, his utility will be

$$\min\{\frac{m}{2\sqrt{p_1 p_2}}, \frac{m'}{2\sqrt{p_3 p_4}}\}$$

This is maximized when

$$\frac{m}{2\sqrt{p_1 p_2}} = \frac{m'}{2\sqrt{p_3 p_4}}$$

or equivalently,

$$\frac{m}{m'} = \frac{\sqrt{p_1 p_2}}{\sqrt{p_3 p_4}} \tag{1}$$

We also must have

$$m + m' = M. \tag{2}$$

From Equations 1 and 2 it follows that

$$m = \frac{\sqrt{p_1 p_2}}{\sqrt{p_1 p_2} + \sqrt{p_3 p_4}} M$$

and

$$m' = \frac{\sqrt{p_3 p_4}}{\sqrt{p_1 p_2} + \sqrt{p_3 p_4}} M.$$

Then

$$x_1 = \frac{m}{2p_1} = \frac{\sqrt{p_2}}{2\sqrt{p_1}(\sqrt{p_1 p_2} + \sqrt{p_3 p_4})} M$$

Then

$$x_2 = \frac{m}{2p_2} = \frac{\sqrt{p_1}}{2\sqrt{p_2}(\sqrt{p_1 p_2} + \sqrt{p_3 p_4})} M$$

Then

$$x_3 = \frac{m'}{2p_3} = \frac{\sqrt{p_4}}{2\sqrt{p_3}(\sqrt{p_1 p_2} + \sqrt{p_3 p_4})} M$$

Then
$$x_4 = \frac{m'}{2p_4} = \frac{\sqrt{p_3}}{2\sqrt{p_3}(\sqrt{p_1p_2} + \sqrt{p_3p_4})}M$$

ii) $U(x_1, x_2, x_3, x_4) = \sqrt{x_1x_2} + \sqrt{x_3x_4}$

As in part i, suppose that he has total income M and spends m on goods 1 and 2 and m' on goods 3 and 4. Then he will choose x_1 and x_2 to maximize $\sqrt{x_1x_2}$ subject to $p_1x_1 + p_2x_2 = m$ and he will choose x_3 and x_4 to maximize $\sqrt{x_3x_4}$ subject to $p_3x_3 + p_4x_4 = m'$. If he does this, his utility will be

$$\frac{m}{2\sqrt{p_1p_2}} + \frac{m'}{2\sqrt{p_1p_2}} \tag{3}$$

He will choose m and m' to maximize Equation3 subject to the constraint that $m + m' = M$. This implies that he will choose $m = M$ and $m' = 0$ if $p_1p_2 < p_3p_4$ and he will choose $m = 0$ and $m' = M$ if this inequality is reversed. So if $p_1p_2 < p_3p_4$, then $x_1 = \frac{M}{2p_1}$, $x_2 = \frac{M}{2p_2}$, $x_3 = x_4 = 0$. If $p_1p_2 > p_3p_4$, then $x_1 = x_2 = 0$, $x_3 = \frac{M}{p_3}$, $x_4 = \frac{M}{p_4}$. I will leave it to you to figure out what happens if $p_1p_2 = p_3p_4$.

3) A pure exchange economy has 1000 type A participants and 1000 type B participants. There are two goods, named goods 1 and 2. Each type A has an initial endowment of 9 units of Good 1 and no Good 2. Each type B has an initial endowment of 16 units of Good 2 and no good 1. Each Type A consumer has the utility function

$$U_A(x_1, x_2) = x_1 + 2\sqrt{x_2}.$$

Each Type B consumer has utility function

$$U_B(x_1, x_2) = x_2 + 2\sqrt{x_1}.$$

Let good 1 be the numeraire and let p be the price of good 2.

i) Find the excess demand function of each type of consumer for good 1. For what prices are there corner solutions where one or both consumers would choose to consume only one good?

At an interior solution, a type A will have $x_2^A = \frac{1}{p^2}$ *and* $x_1^A = 9 - px_2^A = 9 - \frac{1}{p}$. *For Type B we have* $x_1^B = p^2$ *and* $px_2^B = 16p - x_1^B$, *so* $x_2^B = 16 - p$.

Type A will consume a positive amount of both goods if and only if $p > 1/9$. *Type B will consume a positive amount of each good if and only if* $p < 16$. *If* $p \leq 1/9$, *Type A will consume only good 2. If* $p \geq 16$, *Type B will consume only good 1.*

ii) For what price or prices is there a competitive equilibrium in which both consumers consume positive amounts of each good?

If both consumers consume positive amounts of each good, total excess demand for good 2 is

$$1000\left(\frac{1}{p^2} + 16 - p - 16\right) = 1000\left(\frac{1}{p^2} - 1\right)$$

So excess demand is zero only if $p^2 = 1/p$. *The only non-negative p for which this is true is* $p = 1$.

iii) Find all of the competitive equilibria for this economy. (Hint: Pay attention to corner solutions. A really outstanding answer would not only find all of the equilibria but show that there are no others.)

The only possibilities for corner solutions would have $x_2^B(p) = 0$ *and* $x_2^A(p) = 16$ *or* $x_1^A(p) = 0$ *and* $x_1^B(p) = 9$. *(I leave this for you to prove.) If* $x_2^B = 0$, *then it must be that* $p \geq 16$. *But if* $p \geq 16$, *then* $x_2^A(p) = 1/p^2 < 16$. *So there can't be a competitive equilibrium with* $x_2^B = 0$. *If* $x_1^A(p) = 0$, *then* $p \leq 1/9$. *If* $p \leq 1/9$, *then* $x_1^A = 1/p^2 \geq 81 \geq 9$. *So there can't be an equilibrium with* $x_1^A = 0$. *So there are no equilibria with corner solutions and only one equilibrium with an interior solution.*

4) An expected utility maximizer seeks to maximize the expected value of

$$U(W) = \frac{W^\gamma}{\gamma}$$

where $\gamma < 1$.

i) Does this person have increasing, decreasing, or constant relative risk aversion? (To get credit, you must show why your answer is true.)

The index of relative risk aversion is

$$\frac{-WU''(W)}{U'(W)}.$$

In this case,

$$\frac{-WU''(W)}{U'(W)} = \frac{-W(\gamma-1)W^{\gamma-2}}{W^{\gamma-1}} = 1-\gamma.$$

Since $1-\gamma$ is constant with respect to W, the person has constant relative risk aversion.

ii) *Does this person have increasing, decreasing, or constant absolute risk aversion? (To get credit, you must show why your answer is true.)*

The index of absolute risk aversion is

$$\frac{-U''(W)}{U'(W)} = \frac{-(\gamma-1)W^{\gamma-2}}{W^{\gamma-1}} = \frac{1-\gamma}{W}.$$

This is a decreasing function of W and so the person has decreasing absolute risk aversion.

iii) *This person owns a product that has a probability p of failing and if it fails, her loss of wealth will be L. With probability $1-p$ it will not fail and she will have no loss. She is considering whether to purchase a warranty on the product. The warranty costs C and would pay her L if the product fails. Assume that the cost of the warranty exceeds the expected loss pL from product failure. Write an equation for the level of wealth at which she would be just indifferent between buying the warranty and not.*

Let her wealth be W. Then she will be indifferent between buying the insurance and not if

$$pU(W-L) + (1-p)U(W) = U(W-C),$$

which is equivalent to

$$\frac{1}{\gamma}\left(p(W-L)^{\gamma} + (1-p)W^{\gamma}\right) = \frac{1}{\gamma}(W-C)^{\gamma}.$$

iv) Suppose that $\gamma = -1$. For what levels of wealth would she buy the warranty and for what levels would she not?

With $\gamma = 1$, she would prefer buying the insurance if

$$\frac{-1}{W-C} > \frac{-p}{W-L} + \frac{-1-p}{W}.$$

Assuming $W > L$ and $W > C$, this is equivalent to

$$W(W-L) < p((W-C)W + (1-p)(W-C)(W-L).$$

Multiply this out and note that the W^2 terms on both sides cancel. Rearrange the resulting linear inequality to find that it is equivalent to

$$W(C - pL) < (1 - p)CL.$$

Since, by assumption, $C > pL$, this is equivalent to

$$W < \frac{(1 - p)CL}{C - pL}.$$

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5) *True or False: If true, prove it. If false give a counterexample*

i) *Choices of a consumer who chooses consumption bundles so as to maximize a quasi-concave, locally non-satiated utility function will necessarily satisfy the weak axiom of revealed preference.*

False. The weak axiom of revealed preferences says that if x^0 is chosen when $x^1 \neq z^0$ could be afforded, then x^1 will never be chosen at prices where x^0 can be afford. If preferences are quasi-concave, but not strictly quasi-concave, there might be more than one bundle x that maximizes the consumer's preferences subject to a budget constraint. If this is the case let x^0 and x^1 be two different bundles that maximize the consumer's utility subject to the same budget constraint with price vector p.

For example, suppose that we observe two choices made by a consumer who has the utility function $u(x_1, x_2) = x_1 + x_2$. One time he had the price vector $p^0 = (1, 1)$ he chose bundle $x^0 = (2, 1)$. At another time, he faced the same price vector $p^1 = p^0$ and chose the bundle $x^1 = (1, 2)$. Both choices would maximize preferences subject to $u(x_1, x_2) = x_1 + x_2$ if his income is 3. Calculation shows that $p^0x^0 \geq p^0x^1$. If WARP is satisfied, it must then be that $p^1x^1 < p^1x^0$. But calculation shows that $p^1x^1 = p^1x^0$. So WARP is violated.

ii) *Choices of a consumer who chooses consumption bundles so as to maximize a* strictly *quasi-concave, locally non-satiated utility function will necessarily satisfy the weak axiom of revealed preference.*

True.

If u is strictly quasi concave, then if x^0 is chosen at price vector p^0, then $u(x) < u(x^0)$ for all x such that $p^0x \leq p^0x^0$. Suppose that $p^0x^0 \geq p^0x^1$. Then it must be that $u(x^0) > u(x^1)$. Since $u(x^1) > u(x)$ if $x \neq x^1$ and $p^1x \leq p^1x^1$, it must be $p^1x^0 > p^1x^1$.

iii) *Choices of a consumer who chooses consumption bundles so as to maximize a strictly quasi-concave, locally non-satiated utility function will necessarily satisfy the strong axiom of revealed preference.*

True.

Suppose that x^i is chosen at price vector p^i. SARP is satisfied if $p^i x^{i+1} \leq p^i x^i$ for $i = 1, \dots n$ implies that $p^n x^1 > px^n$.

For a consumer who chooses consumption bundles so as to maximize a strictly quasi-concave, locally non-satiated utility function u, it must be that $u(x^{i+1}) < u(x^i)$ for all $i = 2, \dots, n.$ since $u(x^i) > u(x)$ for all x such that $p^i x \leq p^i x^i$. But if $u(x^{i+1}) < u(x^i)$ for $i = 1 \dots n$, then it must be that $u^n(x^n) < u(x^1)$. Since x^n is chosen at prices p^n, it must be that $p^n x^1 > p^n x^n$.

iv) *Choices of a consumer who chooses consumption bundles so as to maximize a strictly quasi-concave, locally non-satiated utility function will necessarily satisfy the generalized axiom of revealed preference.*

True. I will leave this one for you to check.

v) *If there are two goods and a consumer's preferences are defined by the lexicographic order, the consumer's preferences must satisfy the weak axiom of revealed preference and also the strong axiom of revealed preference.*

True. For any budget with positive prices for both goods, the consumer will choose to buy only good 1. It is easy to verify that if there is only one good, the weak and strong axiom are satisfied.

6) *There are two goods, called good 1 and good 2. Mr. Punter is an expected utility maximizer whose preference among lotteries are determined by a von Neumann Morgenstern utility function that is the expected value of*

$$u(x_1, x_2) = x_1^{1/4} x_2^{1/2}.$$

Mr. Punter currently has wealth m_0 and faces prices $p_1 = 1$ and $p_2 = 1$ for goods 1 and 2. He is considering a gamble. If he accepts the gamble, his wealth will be m' and the price of good 1 will be 1, but with probability 1/2, the price of good 2 will be 4 and with probability 1/2, the price of good 2 will be 1. Write an expression for the value of m' that would be just enough to make Mr. Punter indifferent between taking the gamble and not taking it.

Mr. Punter will be indifferent between taking the gamble or not if

$$v(1, 1, m) = \frac{1}{2} v(1, 1, m') + \frac{1}{2} v(1, 4, m')$$

where $v(p,m)$ is the indirect utility function associated with $u(x_1,x_2) = x_1^{1/4}x_2^{1/2}$. Now

$$v(p,m) = \left(\frac{m}{3p_1}\right)^{1/4}\left(\frac{m}{3p_2}\right)^{1/2}$$

Therefore he is indifferent when

$$\frac{2^{1/2}m^{3/4}}{3^{3/4}} = \frac{1}{2}\frac{2^{1/2}m'^{3/4}}{3^{3/4}} + \frac{1}{2}\frac{2^{1/2}m'^{3/4}}{3^{3/4}2}$$

Simplifying this expression, we have

$$m^{3/4} = m'^{3/4}\left(\frac{1}{2}+\frac{1}{4}\right)$$

Therefore

$$m' = m\left(\frac{4}{3}\right)^{4/3}.$$

7) A consumer has utility function $u(x_1, x_2) = \left(x_1^{1/2} + x_2^{1/2}\right)^2$. His income is m^0. We can compare the cost of living for this consumer at two different price vectors, $p^0 = (p_1^0, p_2^0)$ and $p^1 = (p_1^1, p_2^1)$ by means of a cost of living index. Let $v^0 = v(p^0, m^0)$ and $v^1 = v(p^1, m^0)$ be indirect utility, at income and prices p^0 and p^1 respectively. Let the cost of living index be

$$I(p^0, p^1, v^0) = \frac{e(p^1, v^0)}{e(p^0, v^0)}$$

where $e(p, v)$ is the consumer's expenditure function.

i) Find the expenditure function for this consumer and then calculate this cost of living index.

The expenditure function for this consumer is

$$e(p, u) = \left(p_1^{-1} + p_2^{-1}\right)^{-1} u$$

which can also be written as

$$e(p, u) = \frac{p_1 p_2}{p_1 + p_2} u.$$

So you could write the cost of living index as

$$I(p^0, p^1, v^0) = \left(\frac{p_1^0 p_2^0}{p_1^0 + p_2^0}\right) v^0 \div \frac{p_1^1 p_2^1}{p_1^1 + p_2^1} v^0 = \left(\frac{p_1^0 p_2^0}{p_1^1 p_2^1}\right) \left(\frac{p_1^1 + p_2^1}{p_1^0 + p_2^0}\right)$$

ii) Suppose that the consumer has a choice between a price vector $p^0 = (1, 1)$ with income m^0 and a price vector $p^1 = (p_1^1, p_2^1)$ with income m^1. How must the cost of living index be related to m^0 and m^1 for him to prefer the price vector $p^1 = (p_1^1, p_2^1)$ and income m^1 to the price vector $p^0 = (1, 1)$ and income m^0.

He will prefer the gamble if and only if

$$\frac{m'}{m} > I(p^0, p^1, v^0).$$

I will leave it for you to explain why this is true.