Name ________________________________

Final Exam, Economics 210A, December 2013

There are 7 questions. Answer as many as you can... Good luck!

1) Find the indirect utility function and the expenditure function for a consumer with each of the following utility functions.

i) $U(x_1, x_2) = \min\{x_1, x_2\}$

ii) $U(x_1, x_2) = \sqrt{x_1 x_2}$

iii) $U(x_1, x_2) = x_1 + x_2$

iv) $U(x_1, x_2) = \max\{x_1, x_2\}$

2) Find the Marshallian demand function for a consumer with each of the following utility functions: (Hint: You may want to think about making use of a two-stage process, using indirect utility.)

i) $U(x_1, x_2, x_3, x_4) = \min\{\sqrt{x_1 x_2}, \sqrt{x_3 x_4}\}$

ii) $U(x_1, x_2, x_3, x_4) = \sqrt{x_1 x_2} + \sqrt{x_3 x_4}$

3) A pure exchange economy has 1000 type A participants and 1000 type B participants. There are two goods, named goods 1 and 2. Each type A has an initial endowment of 9 units of Good 1 and no Good 2. Each type B has an initial endowment of 16 units of Good 2 and no good 1. Each Type A consumer has the utility function

$$U_A(x_1, x_2) = x_1 + 2\sqrt{x_2}.$$

Each Type B consumer has utility function

$$U_B(x_1, x_2) = x_2 + 2\sqrt{x_1}.$$

Let good 1 be the numeraire and let p be the price of good 2.

i) Find the excess demand function of each type of consumer for good 1. For what prices are there corner solutions where one or both consumers would choose to consume only one good?

ii) For what price or prices is there a competitive equilibrium in which both consumers consume positive amounts of each good?

iii) Find all of the competitive equilibria for this economy. (Hint: Pay attention to corner solutions. A really outstanding answer would not only find all of the equilibria but show that there are no others.)

4) An expected utility maximizer seeks to maximize the expected value of

$$U(W) = \frac{W^\gamma}{\gamma}$$

where $\gamma < 1$.

i) Does this person have increasing, decreasing, or constant relative risk aversion? (To get credit, you must show why your answer is true.)

ii) Does this person have increasing, decreasing, or constant absolute risk aversion? (To get credit, you must show why your answer is true.)

iii) This person owns a product that has a probability p of failing and if it fails, her loss of wealth will be L. With probability $1 - p$ it will not fail and she will have no loss. She is considering whether to purchase a warranty on the product. The warranty costs C and would pay her L if the product fails. Assume that the cost of the warranty exceeds the expected loss pL from product failure. Write an equation for the level of wealth at which she would be just indifferent between buying the warranty and not.

iv) Suppose that $\gamma = -1$. For what levels of wealth would she buy the warranty and for what levels would she not?

5) True or False: If true, prove it. If false give a counterexample

i) Choices of a consumer who chooses consumption bundles so as to maximize a quasi-concave, locally non-satiated utility function will necessarily satisfy the weak axiom of revealed preference.

ii) Choices of a consumer who chooses consumption bundles so as to maximize a *strictly* quasi-concave, locally non-satiated utility function will necessarily satisfy the weak axiom of revealed preference.

iii) Choices of a consumer who chooses consumption bundles so as to maximize a strictly quasi-concave, locally non-satiated utility function will necessarily satisfy the strong axiom of revealed preference.

iv) Choices of a consumer who chooses consumption bundles so as to maximize a quasi-concave, locally non-satiated utility function will necessarily satisfy the generalized axiom of revealed preference.

v) If there are two goods and a consumer's preferences are defined by the lexicographic order, the consumer's preferences must satisfy the weak axiom of revealed preference and also the strong axiom of revealed preference.

6) There are two goods, called good 1 and good 2. Mr. Punter is an expected utility maximizer whose preference among lotteries are determined by a von Neumann Morgenstern utility function that is the expected value of

$$u(x_1, x_2) = x_1^{1/4} x_2^{1/2}.$$

Mr. Punter currently has wealth m_0 and faces prices $p_1 = 1$ and $p_2 = 1$ for goods 1 and 2. He is considering a gamble. If he accepts the gamble, his wealth will be m' and the price of good 1 will be 1, but with probability 1/2, the price of good 2 will be 4 and with probability 1/2, the price of good 2 will be 1. Write an expression for the value of m' that would be just enough to make Mr. Punter indifferent between taking the gamble and not taking it.

7) A consumer has utility function $u(x_1, x_2) = \left(x_1^{1/2} + x_2^{1/2}\right)^2$. His income is m^0. We can compare the cost of living for this consumer at two different price vectors, $p^0 = (p_1^0, p_2^0)$ and $p^1 = (p_1^1, p_2^1)$ by means of a cost of living index. Let $v^0 = v(p^0, m^0)$ and $v^1 = v(p^1, m^0)$ be indirect utility, at income and prices p^0 and p^1 respectively. Let the cost of living index be

$$I(p^0, p^1, v^0) = \frac{e(p^1, v^0)}{e(p^0, v^0)}$$

where $e(p, v)$ is the consumer's expenditure function.

i) Find the expenditure function for this consumer and then calculate this cost of living index.

ii) Suppose that the consumer has a choice between a price vector $p^0 = (1, 1)$ with income m^0 and a price vector $p^1 = (p_1^1, p_2^1)$ with income m^1. How must the cost of living index be related to m^0 and m^1 for him to prefer the price vector $p^1 = (p_1^1, p_2^1)$ and income m^1 to the price vector $p^0 = (1, 1)$ and income m^0. Explain your answer.