Econ 210A Demand Problems, October, 2009

1) An economy has M consumers. For $i = 1, \ldots, M$, Consumer i has utility function

$$U_i(x_{1i}, x_{2i}) = x_{1i} + 2a_i \sqrt{x_{2i}}$$

where x_{ji} is the amount of good j that i consumes and where $a_i > 0$.

a) Find Consumer i's demand function $x^i(p, w_i)$ for Goods 1 and 2. Be sure to take account of possible corner solutions. For what values of p_1, p_2, and w does Consumer i consume none of Good 1?

b) Show that if "everyone has enough income", all consumers will demand positive amounts of both goods. What do we mean by everyone has enough income in this context?

c) Write an expression for consumer i's indirect utility function. Are these indirect utilities of the Gorman form?

d) In general, aggregate demand is the sum of individual demands which in turn depends on prices and the distribution of income. That is aggregate demand is a function

$$D(p, w_1, \ldots, w_M) = \sum_i x^i(p, w_i).$$

Show that with the utility functions in this problem, there is a range of income distributions such that aggregate demand depends only on prices and total income for all income distributions in this range.

2 An economy has M consumers. For $i = 1, \ldots, M$, Consumer i has utility function

$$U_i(x_{1i}, x_{2i}, x_{3i}) = x_{1i} + 4 (x_{2i} x_{3i})^{1/4}$$

where x_{ji} is the amount of good j that i consumes. Answer questions equivalent to parts a-d of Question 1.

    Hint: Break the problem into two pieces. Given the price vector p, suppose that the consumer spends y on goods 2 and 3. How would he allocate this expenditure between goods 2 and 3. Once you know this, you can write an expression for his utility if he spends y on goods 2 and 3 and consumes x_1 units of good 1. You can write then write his utility and his budget constraint in terms of x_1 and y and solve for the optimal choices of x_1 and y, and hence the optimal choices of x_2 and x_3.

3) Find the Marshallian demand function for each of the three goods for a consumer with utility function utility function

$$U(x_1, x_2, x_3) = x_1 + \ln (x_2 + \ln(1 + x_3)).$$

Hint: Start by finding demand functions for those price-income combinations for which there is positive demand for each of the three goods. First solve for the demand for good 3. Note that this depends only on p_2 and p_3. Now solve

for the demand for good 2. Finally, use the budget constraint to solve for the demand for good 1.

If all 3 goods are consumed at prices p and income m, what would be the effect of an increase in income on the demand for good 1? on the demand for good 2? on the demand for good 3?

Looking at the answers that you found, when would you guess that there are corner solutions, where one or more good is not consumed?