An Admissions Quandry

M. Weasel is the Director of Admissions at a college that prides itself on the SAT scores of its students. Alumni have pressured the college to improve its football team. M. Weasel has decided to institute an "affirmative action" policy as follows. The college compares applicants by a relation P^* constructed as follows. All applicants must take the SAT test. Applicants who play football are ranked for their football prowess by the Director of Violent Activites, Coach Cochon. If neither is a football player, and *i* has a higher SAT score than *j*, then iP^*j . For two students *i* and *j* such that *i* is a football player and *j* is not, then iP^*j unless *j*'s SAT score is more than 20 points higher than *i*'s. If both are football players and Coach Cochon ranks *i* higher than *j*, then iP^*j . The college plans to admit 500 students. In general, would it be possible to use the ranking P^* to choose a class in such a way that there are no two students *i* and *j* such that student *j* was admitted and student *i* was not admitted, although iP^*j . (Hint: Is the relation P^* necessarily transitive?)

An alternative approach would be to add 20 points to each football player's SAT score, then admit the 500 students with highest adjusted scores. What reasonable objection could Coach Cochon make to this scheme? Can you suggest a way to improve on this scheme in a way that is likely to improve the football team without reducing the average SAT scores?