

## Building a Model of a Strategic Situation

# 2

If the human mind was simple enough to understand, we'd be too simple to understand it. —EMERSON PUGH

### 2.1 Introduction

THEY SPEAK OF “DELIVERABLES” in the corporate world as the end product that is—well, delivered to a customer. So for those who are trying to understand social phenomena—such as economists, political scientists, and nosy neighbors—or those trying to determine how to behave—such as policymakers, business owners, and teenagers—game theory has two deliverables. First, it provides a framework for taking a complex social situation and boiling it down to a model that is manageable. Second, it provides methods for extracting insights from that model regarding how people do behave or how they should behave. This chapter focuses on using game theory to model a strategic situation; the next chapter begins our journey solving such models.

Human behavior typically occurs in an environment that is highly complex, and this complexity poses a challenge in modeling social phenomena. Deciding on what to put in a model is like trying to pack for college: there's just no way to shove everything you want into that suitcase. In that light, it is useful to distinguish between literal and metaphorical models. A literal model is a model that is descriptively accurate of the real-world setting it is intended to represent. Other than for board games and a few other settings, a literal model of a social situation would be a bloody mess. In contrast, a metaphorical model is a vast simplification—a simplified analogy—of the real-world situation; it is not meant to be descriptively accurate. With a metaphorical model, we try to *simulate* the real world in essential ways, not replicate it. The “essential” ways are those factors thought to be critical to the problem of interest. Factors that are presumed to be secondary are willfully ignored. Most of the models in this book and most of the models constructed to understand social phenomena are metaphorical. Done right, a metaphorical model can yield insights into human behavior that are applicable to much richer and more realistic situations.

Whether literal or metaphorical, game theory offers a scaffolding around which a model can be constructed, and in this chapter we review the two primary types of scaffolding. The *extensive form* is a description of the sequence of choices faced by those involved in a strategic situation, along with what they know when they choose. In Section 2.2, we consider extensive form games of *perfect information*, in which a person always knows what has thus far transpired in the game. Situations with *imperfect information* are described in Section 2.3, and these models allow a person to lack knowledge about what other people have chosen so far. The central concept of a *strategy*

is introduced in Section 2.3, and this concept provides the foundation for describing the *strategic form* of a game in Section 2.4—the second type of scaffolding. Though more abstract than the extensive form, the strategic form is more concise and easier to work with. *Common knowledge* is a concept pertinent to both methods of modeling a strategic situation and is covered in Section 2.5. Common knowledge deals with what a person knows about what others know.

Before we move forward, let me remind you that this chapter is about *building* a game. *Solving* a game will begin with the next chapter, so be prepared for some delayed gratification.

## 2.2 Extensive Form Games: Perfect Information

IN SPITE OF ITS NAME, game theory can deal with some fairly dire subjects, one of which is the criminal activity of kidnapping for ransom. This is a sufficiently serious and persistent problem in some countries—such as Colombia, Mexico, and Russia—that companies have taken out insurance against their executives being held for ransom. Building a model of kidnapping can involve factoring in a great many considerations. The focus of our task, however, is not so much on gaining insight into kidnapping, but on learning how to construct a game-theoretic model.

Because the objective of game theory is to derive implications about behavior, a model should focus on those individuals who have decisions to make. Our attention will accordingly be on the kidnapper, whom we'll call Guy, and the victim's wife, Vivica, who has been contacted to pay ransom. Although the victim (whom we'll name Orlando) is surely affected by what transpires, we are presuming that the victim has no options. In describing the situation, our model should address the following questions: When do Guy and Vivica get to act? What choices are available when they get to act? What do they know

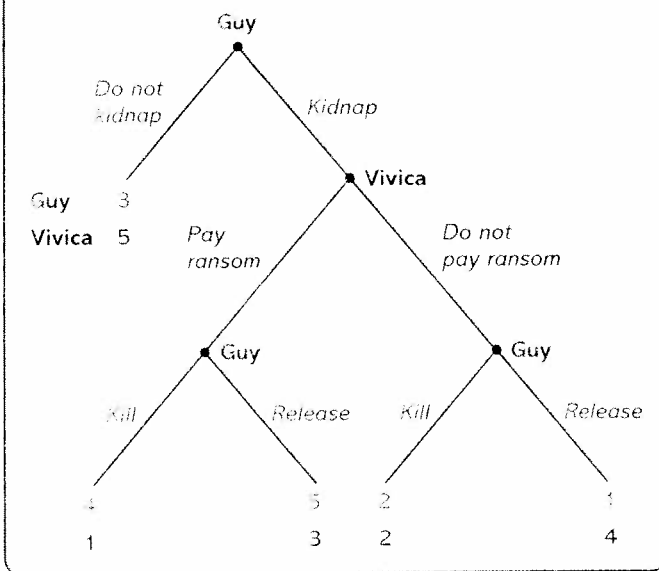
when they get to act? More information will be needed to derive predictions about behavior, but the information obtained by answering these questions is sufficient for starters.

The model is represented by what is known as a **decision tree**, such as that shown in **FIGURE 2.1**. A decision tree is read from top to bottom. (It can also be depicted to be read from left to right.) Each of the dots is called a **decision node**, which represents a point in the game at which someone has a decision to make. Coming out of a decision node is a series of branches, where each **branch** represents a different action available to the decision maker. Choosing a branch is equivalent to choosing an action.

At the top of the decision tree, Guy is to make the initial decision, and his choices are *kidnap* (Orlando) and *do not kidnap*.<sup>2</sup> If he

<sup>2</sup>The name of an action or strategy will typically be *italicized* in this book.

**FIGURE 2.1** Extensive Form of the Kidnapping Game



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chooses the latter, then the tree comes to an end, which represents “game over.” If, instead, he chooses to kidnap Orlando, then Vivica is informed of the kidnapping and decides whether to pay the ransom. In response to Vivica’s decision, Guy decides whether to *release* Orlando or *kill* him. The assumption is that Guy observes whether ransom is paid prior to making this choice. (How to handle a simultaneous exchange will be discussed later in the chapter.)

There is a total of five outcomes to this game, each of which corresponds to a path through the decision tree or, equivalently, a sequence of actions. These outcomes are listed in **TABLE 2.1**. One outcome is for there not to be a kidnapping. If there is a kidnapping, there are four possible outcomes, depending on whether ransom is paid and whether Orlando is killed or released.

**TABLE 2.1 KIDNAPPING GAME AND PAYOFFS**

Outcome	Guy	(Violent) Guy	Vivica
No kidnapping	3	3	5
Kidnapping, ransom is paid, Orlando is killed	4	5	1
Kidnapping, ransom is paid, Orlando is released	5	4	3
Kidnapping, ransom is not paid, Orlando is killed	2	2	2
Kidnapping, ransom is not paid, Orlando is released	1	1	4

The objective of our model is to make some predictions about how Guy and Vivica will behave. Although solving a game won’t be tackled until the next chapter, in fact we don’t have enough information to solve it even if we knew how. To describe how someone will behave, it’s not enough to know what they can do (e.g., *kill* or *release*) and what they know (e.g., whether ransom has been paid); we also need to know what these people care about. What floats their boat? What rings their bell? What tickles their fancy? You get the idea.

A description of what a player cares about takes the form of a ranking of the five outcomes of the game. Suppose Guy is someone who really just wants the money and kills only out of revenge for the ransom not being paid. Then Guy’s best outcome is to perform the kidnapping, Vivica pays the ransom, and he releases Orlando. Because we assume that he is willing to kill in exchange for money, his second-best outcome is to perform the kidnapping, have the ransom paid, and kill Orlando. The third-best outcome is not to kidnap Orlando, since Guy prefers not to run the risk of kidnapping when ransom is not to be paid. Of the two remaining outcomes, suppose that if he kidnaps Orlando and ransom is not paid, then he prefers to kill Orlando (presumably out of spite for not receiving the ransom). The least preferred outcome is then that there is a kidnapping, ransom is not paid, and Orlando is released.

To concisely include Guy’s preferences in our description of the game, we’ll assign a number to each outcome, with a higher number indicating a more preferred outcome for a player. This ranking is done in Table 2.1 under the column labeled “Guy.” These numbers are referred to as **payoffs** and are intended to measure the well-being (or utility, or welfare, or happiness index) of a player. For example, the highest payoff, 5, is assigned to the best outcome: the kidnapping takes place, ransom is paid, and Orlando is released. The worst

outcome—the kidnapping takes place, ransom is not paid, and Orlando is released—receives the lowest payoff, 1.

Suppose, contrary to what was just assumed, that Guy felt that his chances of getting caught would be less if Orlando were dead, so that he now always prefers killing Orlando to releasing him. Then Guy's payoffs would be as shown in the column "(Violent) Guy." The highest payoff is now assigned to the outcome in which Guy kidnaps and kills Orlando and the ransom is paid.

What about Vivica? If she cares about Orlando more than she cares about money, then her most preferred outcome is no kidnapping, and we'll assign that the highest payoff of 5. Her least preferred outcome is that Orlando is kidnapped and killed and ransom is paid, so it receives the lowest payoff of 1. The payoffs for the other outcomes are shown in the table.

To ensure that the depiction in Figure 2.1 contains all of the relevant information, the payoffs have been included. Each terminal node corresponds to a particular outcome of the game, and listed below a terminal node are the payoffs that Guy and Vivica assign to that outcome; the top number is Guy's payoff and the bottom number is Vivica's payoff. While we could also list Orlando's payoffs—he is surely not indifferent about what happens—that would be extraneous information. Because our objective is to say something about behavior, and this model of kidnapping allows only the kidnapper and the victim's kin to act, only their payoffs matter.

This step of assigning a payoff to an outcome is analogous to what was done in Chapter 1. There we began with a person's preferences for certain items (in our example, it was cell phone providers), and we summarized those preferences by assigning a number—known as **utility**—to each item. A person's preferences were summarized by the resulting utility function, and her behavior was described as making the choice that yielded the highest utility. We're performing the same step here, although game theory calls the number a payoff; still, it should be thought of as the same as utility.

The scenario depicted in Figure 2.1 is an example of an *extensive form game*. An **extensive form game** is depicted as a decision tree with decision nodes, branches, and terminal nodes. A decision node is a location in the tree at which one of the players has to act. Let us think about all of the information embodied in Figure 2.1. It tells us which players are making decisions (Guy and Vivica), the sequence in which they act (first Guy then, possibly, Vivica, and then Guy again), what choices are available to each player, and how they evaluate the various outcomes of the game. This extensive form game has four decision nodes: the initial node at which Guy decides whether to kidnap Orlando, the decision node at which Vivica decides whether to pay ransom, and Guy's two decision nodes concerning whether to kill or release Orlando (one decision node for when Vivica pays ransom and one for when she does not). Extending out of each decision node are branches, where a branch represents an action available to the player who is to act at that decision node. More branches mean more choices.

We refer to the decision node at the top of the tree as the **initial node** (that is where the game starts) and to a node corresponding to an end to the game as a **terminal node** (which we have not bothered to represent as a dot in the figure). There are five terminal nodes in this game, since there are five possible outcomes. Terminal nodes are distinct from decision nodes, as no player acts at a terminal node. It is at a terminal node that we list players' payoffs,

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where a payoff describes how a player evaluates an outcome of the game, with a higher number indicating that the player is better off.

► SITUATION: **BASEBALL, I**

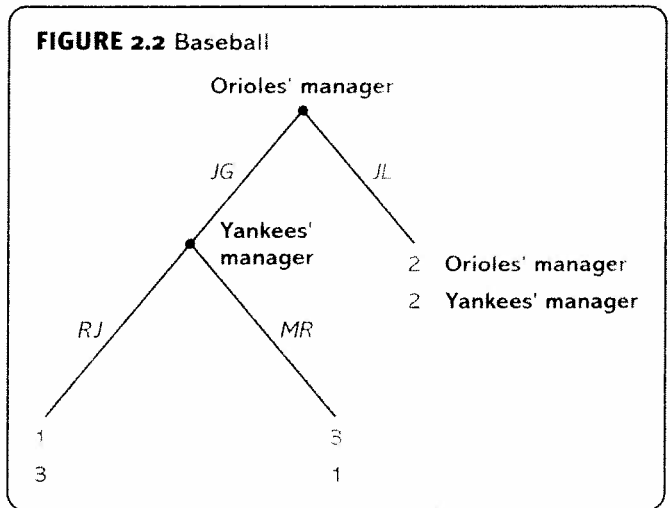
*Good pitching will always stop good hitting and vice-versa.* —CASEY STENGEI

One of the well-known facts in baseball is that right-handed batters generally perform better against left-handed pitchers and left-handed batters generally perform better against right-handed pitchers. **TABLE 2.2** documents this claim.<sup>1</sup> If you're not familiar with baseball, batting average is the percentage of official at bats for which a batter gets a hit (in other words, a batter's success rate). Right-handed batters got a hit in 25.5% of their attempts against a right-handed pitcher, or, as it is normally stated in baseball, their batting average was .255. However, against left-handed pitchers, their batting average was significantly higher, namely, .274. There is an analogous pattern for left-handed batters, who hit .266 against left-handed pitchers but an impressive .291 against right-handed pitching. Let's explore the role that this simple fact plays in a commonly occurring strategic situation in baseball.

Batter	Pitcher	Batting Average
Right	Right	.255
Right	Left	.274
Left	Right	.291
Left	Left	.266

It is the bottom of the ninth inning and the game is tied between the Orioles and the Yankees. The pitcher on the mound for the Yankees is Mariano Rivera, who is a right-hander, and the batter due up for the Orioles is Jay Lopez, who is also a right-hander. The Orioles' manager is thinking about whether to substitute Jay Gibbons, who is a left-handed batter, for Lopez. He would prefer to have Gibbons face Rivera in order to have a lefty-righty matchup and thus a better chance of getting a hit. However, the Yankees' manager could respond to Gibbons pinch-hitting by substituting the left-handed pitcher Randy Johnson for Rivera. The Orioles' manager would rather have Lopez face Rivera than have Gibbons face Johnson. Of course, the Yankees' manager has the exact opposite preferences.

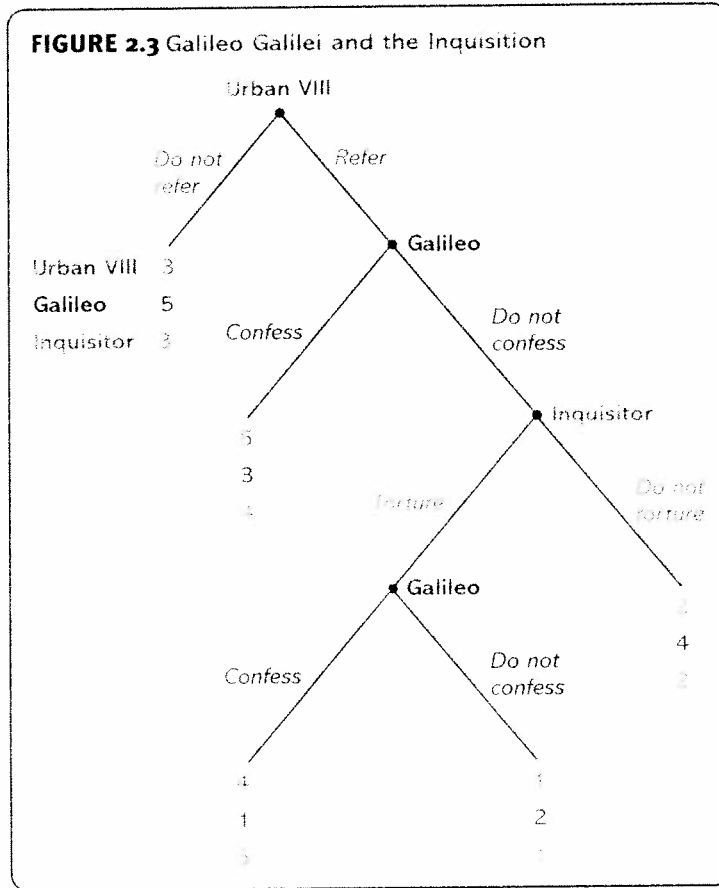
The extensive form of this situation is shown in **FIGURE 2.2**. The Orioles' manager moves first by deciding whether to substitute *Gibbons* for *Lopez*. If he does make the substitution, then the Yankees' manager decides whether to substitute *Johnson* for *Rivera*. Encompassing these preferences, the Orioles' manager assigns the highest payoff (which is 3) to when *Gibbons* bats against *Rivera* and the lowest payoff (1) to when *Gibbons* bats against *Johnson*. Because each manager is presumed to care only about winning, what makes the Orioles better off must make the Yankees worse off. Thus, the best outcome for the Yankees' manager is when *Gibbons* bats against *Johnson*, and the worst is when *Gibbons* bats against *Rivera*.



► SITUATION: GALILEO GALILEI AND THE INQUISITION, I

In 1633, the great astronomer and scientist Galileo Galilei was under consideration for interrogation by the Inquisition. The Catholic Church contends that in 1616 Galileo was ordered not to teach and support the Copernican theory, which is that the earth revolves around the sun, and furthermore that he violated this order with his latest book, *The Dialogue Concerning the Two Chief World Systems*. The situation to be modeled is the decision of the Catholic Church regarding whether to bring Galileo before the Inquisition and, if it does so, the decisions of Galileo and the Inquisitor regarding what to say and do.

The players are Pope Urban VIII, Galileo, and the Inquisitor. (Although there was actually a committee of Inquisitors, we'll roll them all into one player.) The extensive form game is depicted in **FIGURE 2.3**. Urban VIII initially decides whether to *refer* Galileo's case to the Inquisition. If he *declines* to do so, then the game is over. If he does *refer* the case, then Galileo is brought before the Inquisition, at which time he must decide whether to *confess* that he did indeed support the Copernican case too strongly in his recent book. If he *confesses*, then he is punished and the game is over. If he *does not confess*, then the Inquisitor decides whether to *torture* Galileo. If he chooses



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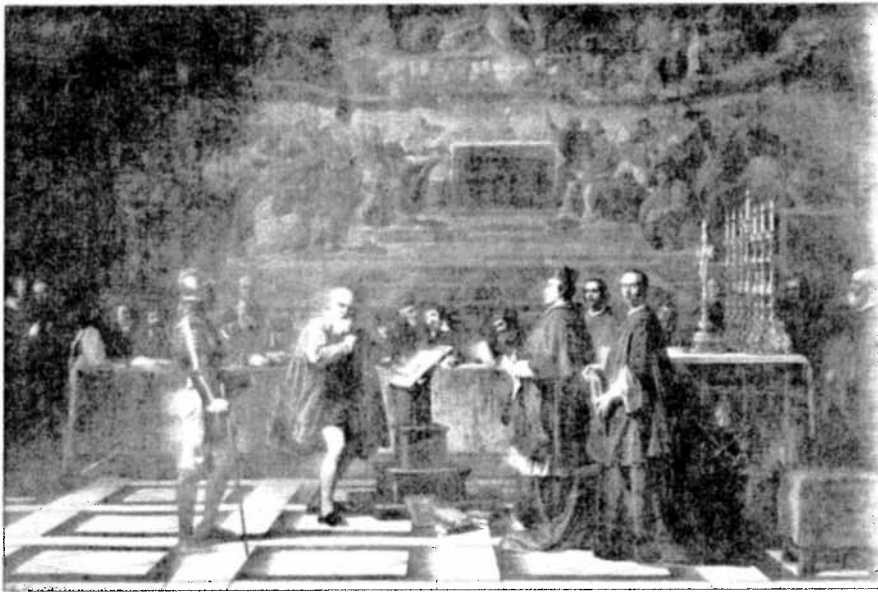


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not to torture him, then, in a sense, Galileo has won, and we'll consider the game ended. If the Inquisitor *tortures* poor Galileo, then he must decide whether to *confess*.

Galileo before the Inquisitor



To complete the extensive form game, payoff numbers are required. There are five outcomes to the game: (1) Urban VIII does not refer the case; (2) Urban VIII refers the case and Galileo initially confesses; (3) Urban VIII refers the case, Galileo does not initially confess, he is tortured, and then he confesses; (4) Urban VIII refers the case, Galileo does not initially confess, he is tortured, and he does not confess; and (5) Urban VIII refers the case, Galileo does not initially confess, and he is not tortured.

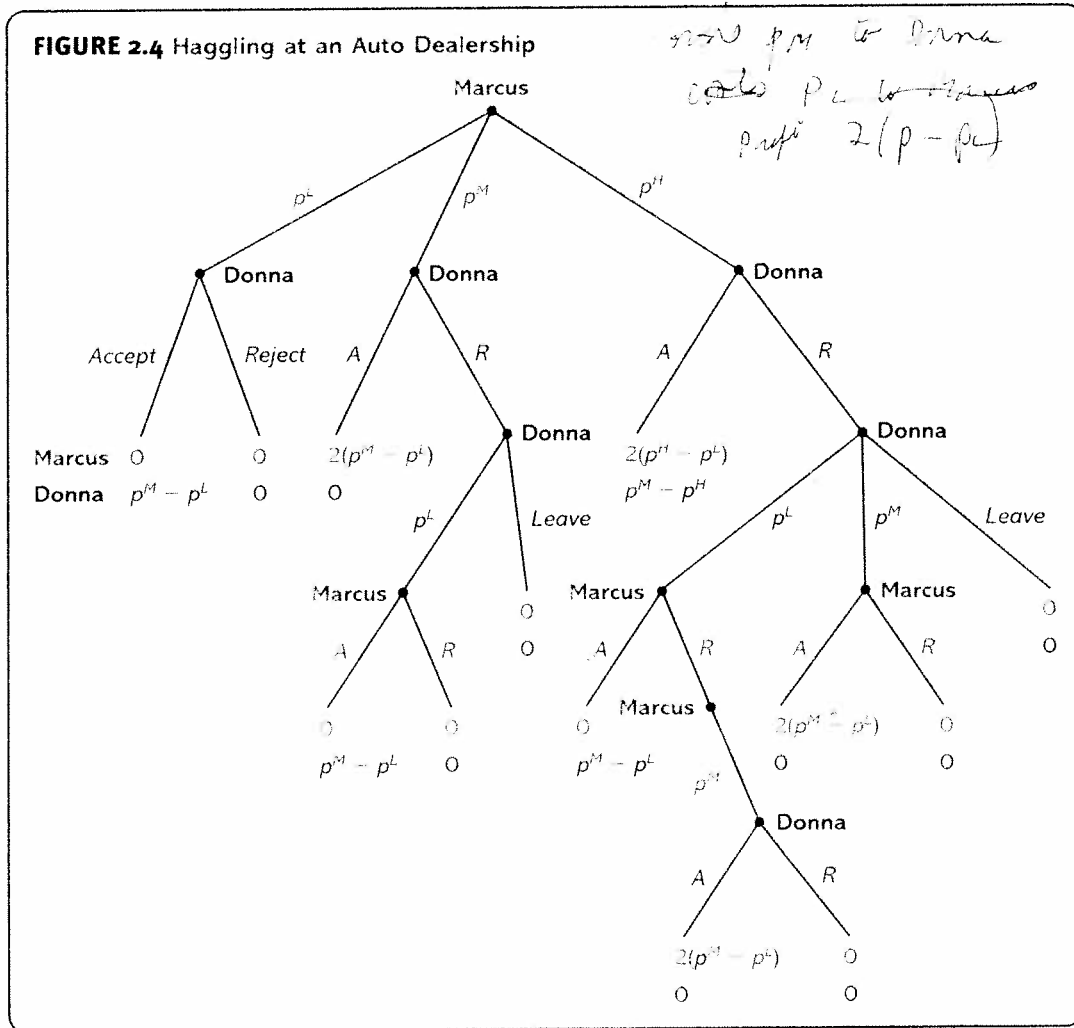
In specifying payoffs, we don't want arbitrary numbers, but rather ones that accurately reflect the preferences of Urban VIII, Galileo, and the Inquisitor. Galileo is probably the easiest. His most preferred outcome is that Urban VIII does not refer the case. We'll presume that if the case is referred, then Galileo's preference ordering is as follows: (1) He does not confess and is not tortured; (2) he confesses; (3) he does not confess, is tortured, and does not confess; and (4) he does not confess, is tortured, and confesses. Galileo was a 69-year-old man, and evidence suggests that he was not prepared to be tortured for the sake of principle. Urban VIII is a bit more complicated, because although he wants Galileo to confess, he does not relish the idea of this great man being tortured. We'll presume that Urban VIII most desires a confession (preferably without torture) and prefers not to refer the case if it does not bring a confession. The Inquisitor's preferences are similar to those of Urban VIII, but he has the sadistic twist that he prefers to extract confessions through torture.

So, what happened to Galileo? Let's wait until we learn how to solve such a game; once having solved it, I'll fill you in on a bit of history.

► SITUATION: HAGGLING AT AN AUTO DEALERSHIP, I

Donna shows up at her local Lexus dealership looking to buy a car. Coming into the showroom and sauntering around a taupe sedan, a salesperson, Marcus, appears beside her. After chatting a bit, he leads the way to his cubicle to negotiate. To simplify the modeling of the negotiation process, suppose the car can be sold for three possible prices, denoted  $p^L$ ,  $p^M$ , and  $p^H$ , and suppose  $p^H > p^M > p^L$ . (*H* is for “high,” *M* is for “moderate,” and *L* is for “low.”)

The extensive form game is depicted in FIGURE 2.4. Marcus initially decides which of these three prices to offer Donna. In response, Donna can either *accept* the offer—in which case the transaction is made at that price—or *reject* it. If it is *rejected*, Donna can either get up and *leave* the dealership (thereby ending the negotiations) or make a *counteroffer*. In the latter case, Donna can respond with a higher price, but that doesn’t make much sense, so it is assumed



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that she selects among those prices which are lower than what she was initially offered (and turned down). For example, if Marcus offers a price of  $p^H$ , then Donna can respond by asking for a price of either  $p^M$  or  $p^L$ . If Donna has decided to *counteroffer*, then Marcus can either *accept* or *reject* her counteroffer. If he *rejects* it, then he can *counteroffer* with a higher price (though it must be lower than his initial offer). This haggling continues until either Donna *leaves*, or an offer is *accepted* by either Donna or Marcus, or they run out of prices to offer.

In terms of payoffs, assume that both Marcus and Donna get a zero payoff if the game ends with no sale. (There is nothing special about zero, by the way. What is important is its relationship to the other payoffs.) If there is a transaction, Marcus's payoff is assumed to be higher when the sale price is higher, while Donna's payoff is assumed to be lower. More specifically, in the event of a sale at a price  $p$ , Donna is assumed to receive a payoff of  $p^M - p$  and Marcus gets a payoff of  $2(p - p^L)$ . (Why multiply by 2? For no particular reason.)

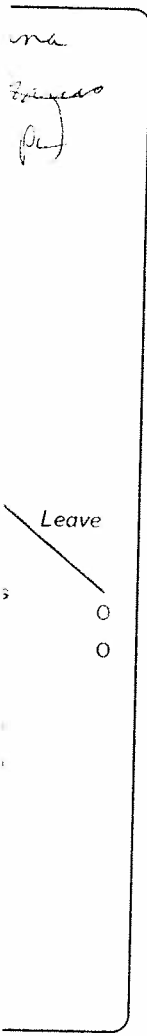
Think about what this is saying. If Marcus sells the car for a price of  $p^L$ , then his payoff is zero because  $2(p^L - p^L) = 0$ . He is then indifferent between selling it for a price of  $p^L$  and not selling the car. At a price of  $p^M$ , his payoff is positive, which means that he's better off selling it at that price than not selling it; and his payoff is yet higher when he sells it for  $p^H$ . For Donna, she is indifferent between buying the car at a price of  $p^M$ , and not buying it, since both give the same payoff (of zero). She prefers to buy the car at a price of  $p^L$ , as that price gives her a payoff of  $p^M - p^L > 0$ ; she is worse off (relative to not buying the car) when she buys it at a price of  $p^H$ , since that gives her a payoff of  $p^M - p^H < 0$ . (Yes, payoffs can be negative. Once again, what is important is the *ordering* of the payoffs.) These payoffs are shown in Figure 2.4.

To be clear about how to interpret this extensive form game, consider what can happen when Marcus initially offers a price of  $p^H$ . Donna can either *accept*—in which case Marcus gets a payoff of  $2(p^H - p^L)$  and Donna gets a payoff of  $p^M - p^H$ —or *reject*. With the latter, she can *leave* or *counteroffer* with either  $p^L$  or  $p^M$ . (Recall that we are allowing her to counteroffer only with a price that is lower than what she has been offered.) If Donna chooses the counteroffer of  $p^L$ , then Marcus can *accept*—resulting in payoffs of zero for Marcus and  $p^M - p^L$  for Donna—or *reject*, in which case Marcus has only one option, which is to *counteroffer* with  $p^M$ , in response to which Donna can either *accept* or *reject* (after which there is nothing left to do). If she instead chooses the counteroffer  $p^M$ , then Marcus can *accept* or *reject* it. If he *rejects*, he has no counteroffer and the game ends.

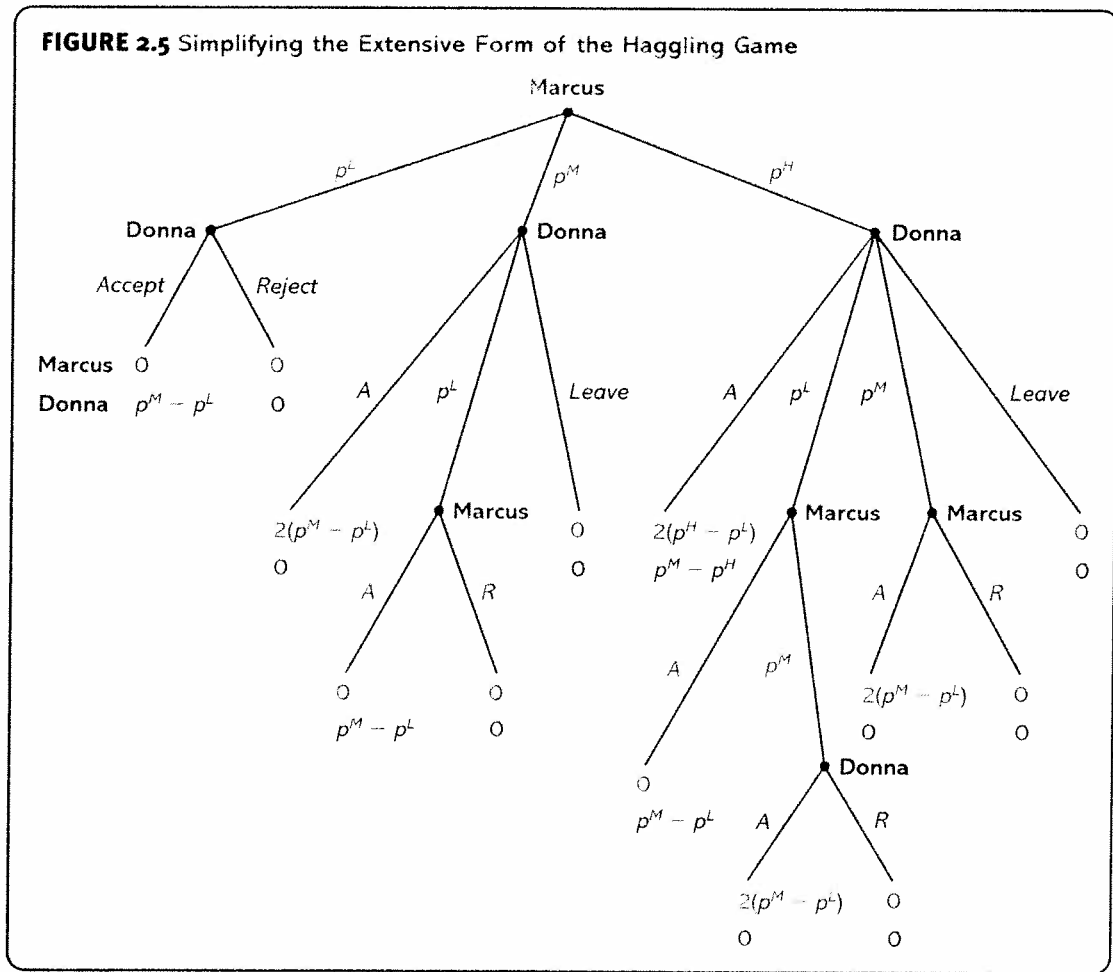
It is worth noting that this extensive form game can be represented alternatively by FIGURE 2.5. Rather than have the same player move twice in a row, the two decision nodes are combined into one decision node with all of the available options. For example, in Figure 2.4, Donna chooses between *accept* and *reject* in response to an initial offer of  $p^M$  from Marcus, and then, if she chooses *reject*, she makes another decision about whether to *counteroffer* with  $p^L$  or *leave*. Alternatively, we can think about Donna having three options (branches) when Marcus makes an initial offer of  $p^M$ : (1) *accept*; (2) *reject* and *counteroffer* with  $p^L$ ; and (3) *reject* and *leave*. Figure 2.5 is a representation equivalent to that in Figure 2.4 in the sense that when we end up solving these games, the same answer will emerge.

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ially decides an either *accept*—or *reject* it, thereby ending—Donna can *reject* it is assumed



**FIGURE 2.5** Simplifying the Extensive Form of the Haggling Game



**2.1 CHECK YOUR UNDERSTANDING\***

Consider a two-player game in which a father chooses between actions *yes*, *no*, and *maybe*. His daughter moves second and chooses between *stay home* and *go to the mall*. The payoffs are as follows:

Outcome	Father's Payoff	Daughter's Payoff
<i>yes</i> and <i>stay home</i>	3	3
<i>yes</i> and <i>go to the mall</i>	5	3
<i>no</i> and <i>stay home</i>	4	1
<i>no</i> and <i>go to the mall</i>	1	5
<i>maybe</i> and <i>stay home</i>	7	2
<i>maybe</i> and <i>go to the mall</i>	2	7

Write down the extensive form game for this strategic situation.

\*All answers to Check Your Understanding are in the back of the book.

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## 2.3 Extensive Form Games: Imperfect Information

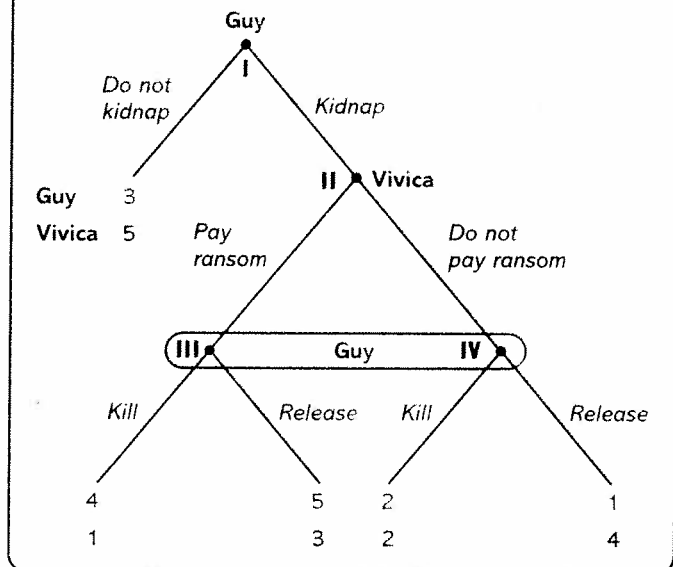
RETURNING TO THE KIDNAPPING SCENARIO, suppose we want to model Guy (the kidnapper) and Vivica (the victim's kin) as making their decisions without knowledge of what the other has done. The extensive form game in Figure 2.1 assumes that Guy learns whether ransom has been paid prior to deciding what to do with Orlando (the victim). An alternative specification is that Guy decides what to do with Orlando at the same time that Vivica decides about the ransom. You could imagine Guy deciding whether to release Orlando somewhere in the city while Vivica is deciding whether to leave the ransom at an agreed-upon location. How do you set up an extensive form game with that feature?

The essential difference between these scenarios is information. In Figure 2.1, Guy knew what Vivica had done when it was time for him to make his decision, whereas Vivica did not know what was to happen to Orlando when she had to decide about paying the ransom. Vivica's lack of knowledge was represented by having Vivica move before Guy. Now we want to suppose that at the time he has to decide about killing or releasing Orlando, Guy is also lacking knowledge about what Vivica is to do or has done. Well, we can't make a decision tree in which Vivica moves after Guy and Guy moves after Vivica.

To be able to represent such a situation, the concept of an *information set* was created. An **information set** is made up of all of the decision nodes that a player is incapable of distinguishing among. Every decision node belongs to one and only one information set. A player is assumed to know which information set he is at, but nothing more. Thus, if the information set has more than one node, then the player is uncertain as to where exactly he is in the game. All this should be clearer with an example.

FIGURE 2.6 is a reformulation of the Kidnapping game with the new assumption that Guy doesn't get to learn whether Vivica has paid the ransom when he decides what to do with Orlando. In terms of nodes and branches, the trees in Figures 2.1 and 2.6 are identical. The distinctive element is the box drawn around the two decision nodes associated with Guy choosing whether to release or kill Orlando (which are denoted III and IV). The nodes in that box make up Guy's information set at the time he has to decide what to do with Orlando. Guy is assumed to know that the game is at either node III or node IV, but that's it; he doesn't know which of the two it is. Think about what this means. Not to know whether the game is at node III or node IV means that Guy doesn't know whether the sequence of play has been "kidnap and ransom is paid" or "kidnap and ransom is not paid." Well, this is exactly what we wanted to model; Guy doesn't know whether

**FIGURE 2.6** Kidnapping Game When the Exchange Is Simultaneous. The Box Around Nodes III and IV Represents the Information Set at the Point That Guy Has to Decide What to Do with Orlando

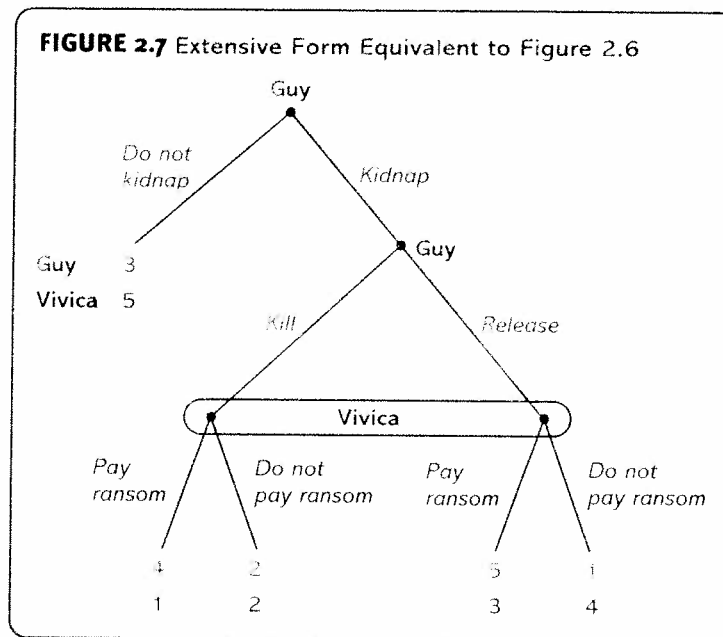


the ransom is to be paid when he must decide whether to release or kill Orlando. The way this situation is represented is that Guy doesn't know exactly where he is in the game: Is he at node III or node IV?

In any extensive form game, a player who is to act always has an information set representing what he knows. So what about when Vivica moves? What is her information set? It is just node II; in other words, she knows exactly where she is in the game. If we want to be consistent, we would then put a box around node II to represent Vivica's information set. So as to avoid unnecessary clutter, however, singleton information sets (i.e., an information set with a single node) are left unboxed. In Figure 2.6, then, Guy has two information sets; one is the singleton composed of the initial node (denoted I), and the other comprises nodes III and IV.

Returning to Vivica, since she is modeled as moving before Guy decides about Orlando, she makes her decision without knowing what has happened or will happen to Orlando. Do you notice how I'm unclear about the timing? Does Vivica move chronologically before, after, or at the same time as Guy? I've been intentionally unclear because it doesn't matter. What matters is *information*, not the time of day at which someone makes a decision. What is essential is that Vivica does not know whether Guy has released or killed Orlando when she decides whether to pay ransom and that Guy does not know whether Vivica has paid the ransom when he decides whether to release or kill Orlando. In fact, **FIGURE 2.7** is an extensive form game equivalent to that in Figure 2.6. It flips the order of decision making between Vivica and Guy, and the reason it is equivalent is that we haven't changed the information that the players have when they move. In both games, we'll say that Vivica and Guy move simultaneously (with respect to the ransom and release-or-kill decisions), which is meant to convey the fact that their

**FIGURE 2.7** Extensive Form Equivalent to Figure 2.6



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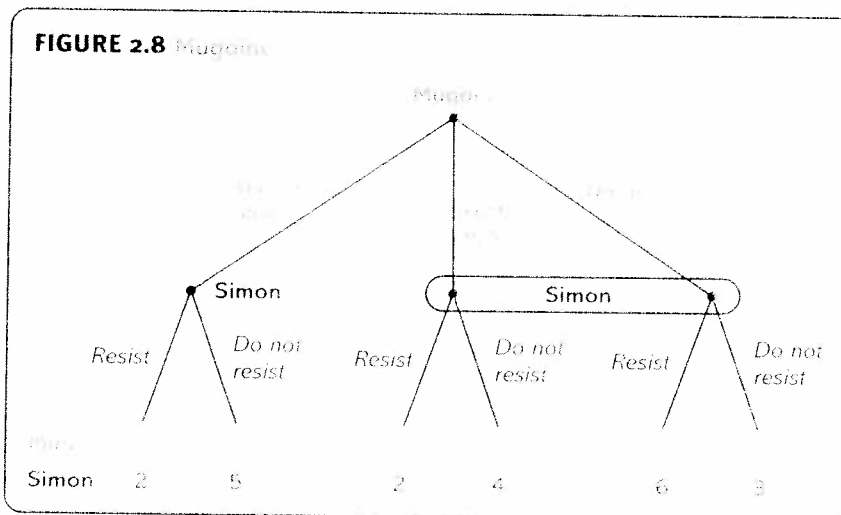
An extensive form game in which all information sets are singletons, such as the games in Figures 2.1–2.5—is referred to as a game of **perfect information**, since players always know where they are in the game when they must decide. A game in which one or more information sets are not singletons, such as the game in Figure 2.6, is known as a game of **imperfect information**.

**FIGURE 2.8 MUGGING**

*Notorious for being cheap, the comedian Jack Benny would tell the following story: "I was walking down a dark alleyway when someone came from behind me and said, 'Your money or your life.' I stood there frozen. The mugger said again, 'Your money or your life.' I replied, 'I'm thinking. . . . I'm thinking.'"*

Simon is walking home late at night when suddenly he realizes that there is someone behind him. Before he has a chance to do anything, he hears, "I have a gun, so keep your mouth shut and give me your wallet, cell phone, and iPod." Simon doesn't see a gun, but does notice that the mugger has his hand in his coat pocket, and it looks like there may be a gun in there. If there is no gun, Simon thinks he could give the mugger a hard shove and make a run for it. But if there is a gun, there is a chance that trying to escape will result in him being shot. He would prefer to hand over his wallet, cell phone, and even his iPod than risk serious injury. Earlier that evening, the mugger was engaging in his own decision making as he debated whether to use a gun. Because the prison sentence is longer when a crime involves a gun, he'd really like to conduct the theft without it.

The mugging situation just described is depicted as the extensive form game in **FIGURE 2.8**. The mugger moves first in deciding between three options: not to use a gun; bring a gun, but not show it to the victim; and bring a gun



and show it to the victim. In response to each of these actions, Simon has to decide whether to resist the mugger by doing the "shove and run" (*resist*) or by complying with the mugger's instructions (*do not resist*). Simon has two information sets. One is a singleton and is associated with the mugger's having and showing a gun. The other information set comprises two nodes, one corresponding to the mugger's having a gun, but not showing it, and the other to the mugger's not having a gun. With the latter information set, Simon isn't sure whether the mugger's pocket contains a gun.

In specifying the payoffs, the best outcome for Simon is that the mugger does not use a gun and Simon resists; the worst outcome is that the mugger has a gun and Simon resists. For the mugger, the best outcome is that Simon does not resist and the mugger doesn't use a gun in the robbery. The worst outcome is that he doesn't use the gun and Simon resists, as then the mugger comes away empty handed.

► SITUATION: **U.S. COURT OF APPEALS FOR THE FEDERAL CIRCUIT**

When the U.S. Court of Appeals for the Federal Circuit hears a case, a panel of 3 judges is randomly selected from the 12 judges on the court. After a case is filed, the parties submit written briefs stating their argument. If the court decides to hear oral arguments, each party's lawyer is given between 15 and 30 minutes. The panel of 3 judges then decides the case. Let us model a simplified version of this judicial setting when there is no oral argument.

One side of the case is represented by attorney Elizabeth Hasenpfeffer, while attorney Joseph Fargullio represents the other party. Prior to their appearance, each attorney decides on a legal strategy and writes a brief based on it. For Ms. Hasenpfeffer, let us denote the strategies as *A* and *B*; for Mr. Fargullio, they'll be denoted *I* and *II*. The briefs are submitted simultaneously, in the sense that each attorney writes a brief not knowing what the other has written. This situation is reflected in **FIGURE 2.9**, in which Ms. Hasenpfeffer moves first and Mr. Fargullio moves second, but with an information set that encompasses both the node in which Ms. Hasenpfeffer chose *A* and the one in which she chose *B*.

After reading the two briefs, the three members of the court then vote either in favor of Ms. Hasenpfeffer's argument or in favor of Mr. Fargullio's argument. This vote is cast simultaneously in that each judge writes down a decision on a piece of paper. For brevity, the judges are denoted *X*, *Y*, and *Z*. As depicted, each judge has four information sets, where an information set corresponds to the pair of legal strategies selected by the attorneys. Judge *X* moves first and thus doesn't know how judges *Y* and *Z* have voted. Judge *Y* moves second and thus doesn't know how Judge *Z* has voted (since *Z* is described as moving after him), but she also doesn't know how Judge *X* has voted because of the structure of the information sets. Each of Judge *Y*'s information sets includes two decision nodes: one for Judge *X* voting in favor of Ms. Hasenpfeffer and one for Judge *X* in favor of Mr. Fargullio. Turning to Judge *Z*, we see that each of her information sets comprises the four nodes that correspond to the four possible ways that Judges *X* and *Y* could have

FIGURE 2.9





voted. Although the judges are depicted as moving sequentially, in fact each votes without knowledge of how the other two have voted; in other words, the judges vote simultaneously.

► **SITUATION: THE IRAQ WAR AND WEAPONS OF MASS DESTRUCTION**

Now let's take on some recent history: the situation faced by Iraq, the United Nations, and the United States that culminated in the U.S. invasion of Iraq on March 20, 2003. At issue is whether Sadaam Hussein has weapons of mass destruction (WMD). As shown in **FIGURE 2.10**, Iraq is modeled as having a choice of possessing or not possessing WMD. Without knowledge of Iraq's choice, the United Nations decides whether to request inspections of Iraq. The United Nations then has one information set, which includes both of Iraq's feasible actions: the one when it has WMD and the other when it does not. If the United Nations chooses not to request inspections, then the United States decides whether or not to invade Iraq, at which point we'll consider the game done. If the United Nations does request inspections, then the move goes back to Iraq. If Iraq does not have WMD, then it can choose to deny inspections or allow them. If, instead, Iraq has WMD, then it can deny inspections, allow inspections, or allow inspections and hide the WMD. With the last option, suppose Iraq succeeds in preventing inspectors from finding WMD. Assume that when Iraq does have WMD and does not hide them from the inspectors, the WMD are found. After Iraq moves in response to the request for inspections by the United Nations, and the outcome of the inspections is revealed, the United States moves again regarding whether to attack Iraq.

The United States has four information sets. The information set denoted I includes the two nodes associated with (1) Iraq having WMD and the United Nations not choosing inspections, and (2) Iraq not having WMD and the United Nations not choosing inspections. Although the United States doesn't get to observe Iraq's choice, it does get to observe the UN decision.

Information set II corresponds to the scenario in which inspections are requested by the United Nations and allowed by Iraq, but WMD are not found, either because Iraq does not have them or because it does have them but has successfully hidden them from the inspectors. Information set III denotes the situation in which the United Nations requests inspections, but they are refused by Iraq; once again, the United States doesn't know whether Iraq has WMD. The lone singleton information set for the United States is node IV,

which is associated with Iraq's having WMD, the United Nation's having requested inspections, and Iraq's having allowing unobstructed inspections, in which case WMD are found. A similar exercise can be conducted to describe the one information set of the United Nations and the three information sets of Iraq (all of which are singletons, as Iraq is the only one hiding something).

## 2.2 CHECK YOUR UNDERSTANDING

Now let us return to the Mugging game and suppose that the mugger not only chooses whether to use a gun and whether to show it, but also whether to load the gun with bullets. If Simon sees the gun, he doesn't know whether it is loaded. Write down the extensive form of this strategic situation. (You can ignore payoffs.)

**FIGURE 2.10** Iraq War and WMD. The Abbreviations W and NW Represent War and No War, Respectively

Iraq

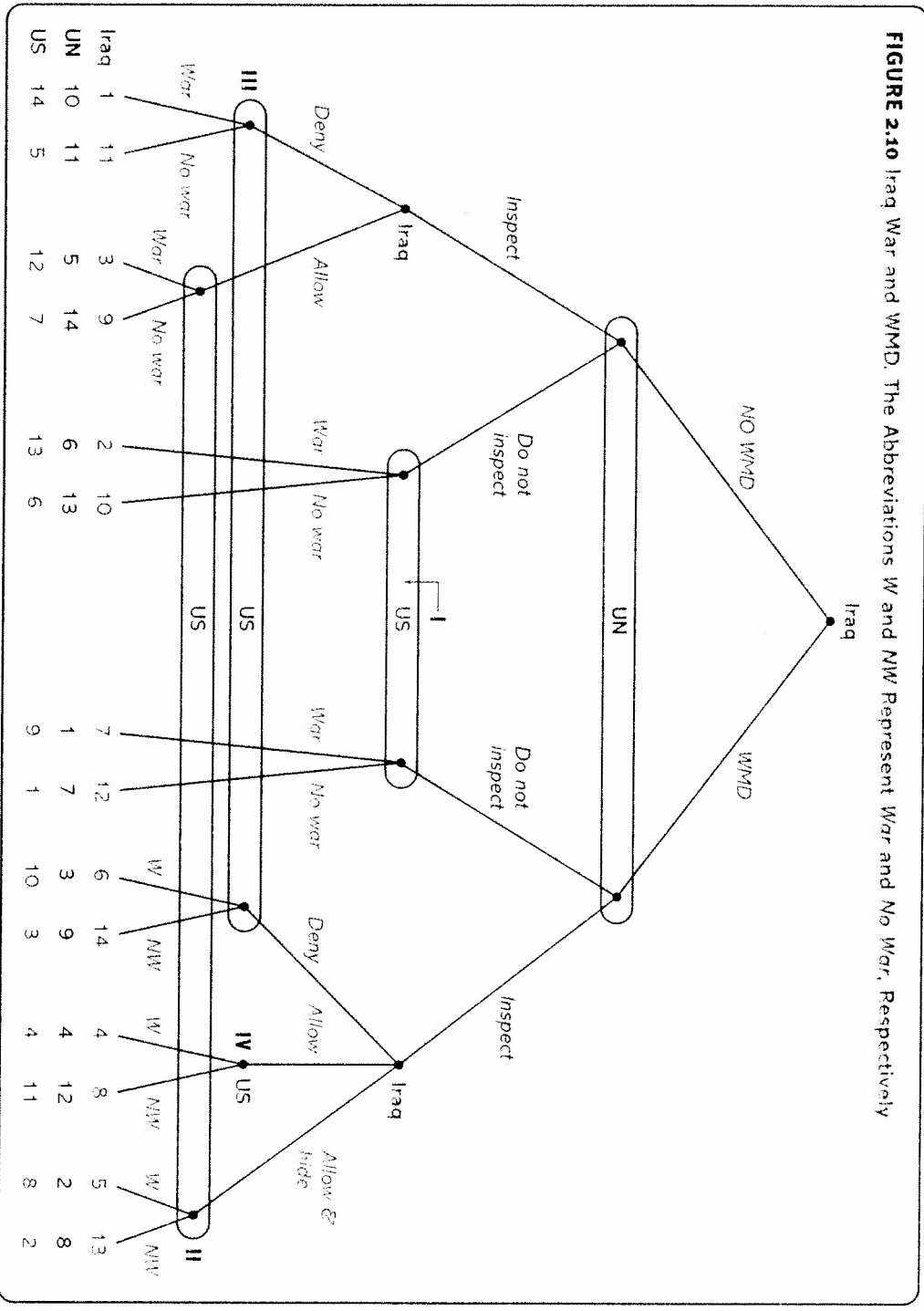


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**FIGURE 2.10** Iraq War and WMD. The Abbreviations *W* and *NW* Represent War and No War, Respectively



## 2.4 What Is a Strategy?

*Victorious warriors win first and then go to war, while defeated warriors go to war first and then seek to win. —Sun Tzu<sup>2</sup>*

WHAT DO YOU THINK the preceding quote from Sun Tzu means? One interpretation is that, to be victorious, you should develop a detailed plan prior to going to battle and then, once in battle, execute that plan. Rather than trying to figure out a plan over the course of the battle, perform all of your thinking before one arrow is flung or one cannon is fired.

The notion of a *strategy* is central to game theory, and its definition is exactly what Sun Tzu had in mind. A **strategy** is a fully specified decision rule for how to play a game. It is so detailed and well specified that it accounts for every contingency. It is not a sequence of actions, but rather a catalog of contingency plans: what to do, depending on the situation. As was well expressed by J. D. Williams in an early book on game theory, a strategy is “a plan so complete that it cannot be upset by enemy action or Nature; for everything that the enemy or Nature may choose to do, together with a set of possible actions for yourself, is just part of a description of the strategy.”<sup>3</sup>

As a conceptual device, we imagine a player choosing a strategy before the game begins. This strategy could, in principle, be written down as a set of instructions and given to another person to play. In other words, having a strategy means doing all of the hard thinking (utilizing intelligence, judgment, cleverness, etc.) prior to playing the game. The actual play is nothing more than following the instructions provided by the strategy selected. Of course, this description of a strategy is an abstraction, since, in practice, surely judgment and acumen are applied in the midst of a strategic situation. However, you’ll need to accept this definition of *strategy* if you are to make headway into gaining insight into strategic situations. It is one of those basic postulates that is valuable in practice because of its purity in form.

To be more concrete as to the nature of a strategy in game theory, let us return to the Kidnapping game in Figure 2.1. What is a strategy for the kidnapper? As we’ve just said, a strategy is a complete decision rule—one that prescribes an action for every situation that a player can find himself in. Guy (the kidnapper) can find himself in three situations: (1) contemplating whether to kidnap Orlando (i.e., the initial node); (2) having kidnapped Orlando, with ransom having been paid by Vivica, and deciding whether to kill or release Orlando; and (3) having kidnapped Orlando, with ransom not having been paid by Vivica, and deciding whether to kill or release Orlando. It is not coincidental that Guy can find himself in *three* scenarios and he has *three* information sets: A “situation” for a player is defined as finding himself at an information set; hence, a strategy assigns one action to each of a player’s information sets.

A template for Guy’s strategy is, then,

At the initial node, \_\_\_\_\_ [fill in *kidnap* or *do not kidnap*].

If a kidnapping occurred and ransom was paid, then \_\_\_\_\_ [fill in *kill* or *release*].

If a kidnapping occurred and ransom was not paid, then \_\_\_\_\_ [fill in *kill* or *release*].

There are as many strategies as ways in which to fill in those three blanks. Exhausting the possibilities, we have eight feasible strategies:

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1. At the initial node, *kidnap*.  
If a kidnapping occurred and ransom was paid, then *release*.  
If a kidnapping occurred and ransom was not paid, then *kill*.
2. At the initial node, *kidnap*.  
If a kidnapping occurred and ransom was paid, then *release*.  
If a kidnapping occurred and ransom was not paid, then *release*.
3. At the initial node, *kidnap*.  
If a kidnapping occurred and ransom was paid, then *kill*.  
If a kidnapping occurred and ransom was not paid, then *release*.
4. At the initial node, *kidnap*.  
If a kidnapping occurred and ransom was paid, then *kill*.  
If a kidnapping occurred and ransom was not paid, then *kill*.
5. At the initial node, *do not kidnap*.  
If a kidnapping occurred and ransom was paid, then *release*.  
If a kidnapping occurred and ransom was not paid, then *kill*.
6. At the initial node, *do not kidnap*.  
If a kidnapping occurred and ransom was paid, then *release*.  
If a kidnapping occurred and ransom was not paid, then *release*.
7. At the initial node, *do not kidnap*.  
If a kidnapping occurred and ransom was paid, then *kill*.  
If a kidnapping occurred and ransom was not paid, then *release*.
8. At the initial node, *do not kidnap*.  
If a kidnapping occurred and ransom was paid, then *kill*.  
If a kidnapping occurred and ransom was not paid, then *kill*.

Analogously, we can define a strategy template for Vivica:

If a kidnapping occurred, then \_\_\_\_\_. [fill in *pay ransom* or *do not pay ransom*].

Since Vivica has just one information set, her strategy is just a single action. With only two feasible actions and one information set, she then has two feasible strategies:

1. If a kidnapping occurred, then *pay ransom*.
2. If a kidnapping occurred, then *do not pay ransom*.

The **strategy set** for a player is defined to be the collection of all feasible strategies for that player. In this example, the strategy set for Guy comprises the eight strategies just listed for him, and the strategy set for Vivica is made up of two strategies. There are then 16 possible strategy pairs for this game.

As previously mentioned, all of the hard thinking goes into choosing a strategy, and once one is chosen, play arises from the implementation of that strategy. To see this point more clearly, suppose Guy chooses the following strategy:

- At the initial node, *kidnap*.
- If a kidnapping occurred and ransom was paid, then *release*.  
If a kidnapping occurred and ransom was not paid, then *kill*.

Suppose also that Vivica chooses the following strategy:

- If a kidnapping occurred, then *pay ransom*.

So what will happen? According to Guy's strategy, he kidnaps Orlando. Vivica then pays the ransom (as instructed by her strategy), and in response to the

ransom being paid, Guy releases Orlando (reading from his strategy). Similarly, you can consider any of the 16 possible strategy pairs and figure out what the ensuing sequence of actions is. It's just a matter of following instructions.

Before moving on, notice a peculiar feature about some of Guy's strategies, namely, that strategies 5 through 8 prescribe *do not kidnap* and then tell Guy what to do if he chose *kidnap*. In other words, it tells him to do one thing, but also what to do if he doesn't do what he should have done. In spite of how

### 2.3 CHECK YOUR UNDERSTANDING

For the revised Kidnapping game in Figure 2.6, write down the strategy sets for Guy and Vivica.

strange that might sound, we'll allow for this possibility in a player's strategy set, for three reasons. First, it's simpler to define a strategy as any way in which to assign feasible actions to information sets than to try to come up with a more complicated definition that rules out these "silly" strategies. Second, inclusion of the silly strategies is, at worst, some harmless detritus that won't affect the conclusions that we draw. And the third reason, which is the most important, I can't tell you now. It's not that I don't want to, but you'll need to know a bit about solving games before you can understand what I want to say. I'll clue you in come Chapter 4.

## 2.5 Strategic Form Games

THE EXTENSIVE FORM IS one type of scaffolding around which a game can be constructed. Its appeal is that it describes (1) a concrete sequence with which players act, (2) what actions they have available and what they know, and (3) how they evaluate the outcomes, where an outcome is a path through the decision tree. In this section, we introduce an alternative scaffolding that, though more abstract, is easier to work with than the extensive form. In the next section, we'll show how you can move back and forth between these two game forms so that you may work with either one.

A **strategic form game** (which, in the olden days of game theory, was referred to as the *normal form*) is defined by three elements that address the following questions: (1) Who is making decisions? (2) Over what are they making decisions? and (3) How do they evaluate different decisions? The answer to the first question is the set of players, the answer to the second question is the players' strategy sets, and the answer to the third question is players' payoff functions.

The **set of players** refers to the collection of individuals who have decisions to make. The decision is with regard to a strategy, which is defined exactly as in the previous section. A player's **strategy set** is the collection of strategies from which he can choose. Finally, a player's **payoff function** tells us how the player evaluates a **strategy profile**, which is a collection of strategies, one for each of the players. A higher payoff means that a player is better off, and when we get to solving a game, the presumption will be that each player tries to maximize his or her payoff.

Although a player does not intrinsically value strategies—for they are just decision rules, and you can't eat, wear, caress, or live in a decision rule—a strategy profile determines the outcome of the game (e.g., whether there is a kidnapping), and a player *does* care about the outcome. One final piece of jargon before we move on: The term ***n*-tuple** refers to *n* of something—for example, an *n*-tuple of strategies in a game with *n* players. Two of something is a pair, three of something is a triple, and *n* of something is an *n*-tuple. With all of this jargon, you can now talk like a game theorist!

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► SITUATION: **TOSCA**

*The force of my desire has two aims, and the rebel's head is not the more precious. Ah, to see the flames of those victorious eyes smoulder, aching with love! Caught in my arms, smouldering with love. One to the gallows, the other in my arms! —BARON SCARPIA FROM THE OPERA TOSCA*

Giacomo Puccini was arguably the last great operatic composer. He died in 1924 after a career that produced such spectacular successes as *La Bohème* (the plot of which was recycled for the Broadway musical *Rent*), *Madame Butterfly*, and *Turandot*. Puccini's music is the type that leads you to hum or whistle it after you leave the theater. It clearly runs counter to the popular definition of opera as two heavy-set people 6 inches apart screaming at the top of their lungs.

One of his most popular operas is *Tosca*, which is a story of love, devotion, corruption, lechery, and murder—in other words, perfect fodder for learning game theory!<sup>4</sup> The main characters are Baron Vitellio Scarpia, the local chief of police; an attractive woman named Floria Tosca; and Mario Cavaradossi, her lover. Scarpia has lustful designs on Tosca and has devised a diabolical plot to act on them. He first has Cavaradossi arrested. He then tells Tosca that Cavaradossi is to go before the firing squad in the morning and he (Scarpia) can order the squad to use real bullets—and Cavaradossi will surely die—or blanks—in which case Cavaradossi will survive. After then hearing Scarpia's sexual demands, Tosca must decide whether or not to concede to them.

Scarpia and Tosca meet that evening after Scarpia has already given his orders to the firing squad. Tosca faces Scarpia and—knowing *that* Scarpia has decided, but not knowing *what* he has decided—chooses between consenting to his lustful desires or thrusting the knife she has hidden in her garments into the heart of this heartless man.

In writing down the strategic form game, we have our two players, Scarpia and Tosca. The strategy set for Scarpia has two strategies—use *real* bullets or use *blanks*—while Tosca can either *consent* or *stab* Scarpia. As depicted in **FIGURE 2.11**, the two strategies for Tosca correspond to the two rows, while the two strategies for Scarpia correspond to the two columns. Thus, Tosca's choosing a strategy is equivalent to her choosing a row.

The final element to the strategic form game are the payoffs. The first number in a cell is Tosca's payoff and the second number is Scarpia's payoff. (We will use the convention that the row player's payoff is the first number in a cell.) For example, if Tosca chooses *stab* and Scarpia chooses *blanks*, then Tosca's payoff is 4 and Scarpia's payoff is 1. We have chosen the payoffs so that Tosca ranks the four possible strategy pairs as follows (going from best to worst): *stab* and *blanks*, *consent* and *blanks*, *stab* and *real*, and *consent* and *real*. Due to her love for Cavaradossi, the most important thing to her is that Scarpia use blanks, but it is also the case that she'd rather kill him than consent to his lascivious libido. From the information in the opening quote, Scarpia's payoffs are such that his most preferred strategy pair is *consent* and *real*, as he then gets what he wants from Tosca and eliminates Cavaradossi as a future rival for Tosca. His least preferred outcome is, not surprisingly, *stab* and *blanks*.

**FIGURE 2.11** *Tosca*

		Scarpia	
		<i>Real</i>	<i>Blanks</i>
Tosca	<i>Stab</i>	2, 2	4, 1
	<i>Consent</i>	1, 4	3, 3

Figure 2.11 is known as a payoff matrix and succinctly contains all of the elements of the strategic form game. *Tosca* is a reinterpretation of the Prisoners' Dilemma, which is the most famous game in the entire kingdom of game theory. I'll provide the original description of the Prisoners' Dilemma in Chapter 4.

► SITUATION: COMPETITION FOR ELECTED OFFICE

*The word 'politics' is derived from the word "poly," meaning "many," and the word "ticks," meaning "blood sucking parasites." —LARRY HARDIMAN*

Consider the path to the U.S. presidency. The Republican and Democratic candidates are deciding where to place their campaign platforms along the political spectrum that runs from liberal to conservative. Let's suppose that the Democratic candidate has three feasible platforms: *liberal*, *moderately liberal*, and *moderate*. Let's suppose that the Republican candidate has three as well: *moderate*, *moderately conservative*, and *conservative*.

A candidate's payoffs are assumed to depend implicitly on the candidate's ideological preferences—what platform he would like to see implemented—and what it'll take to have a shot at getting elected. Assume that most voters are moderate. The Democratic candidate is presumed to be liberal (i.e., his most preferred policies to implement are liberal), but he realizes that he may need to choose a more moderate platform in order to have a realistic chance of winning. Analogously, the Republican candidate is presumed to be conservative, and she, too, knows that she may need to moderate her platform. The payoff matrix is shown in FIGURE 2.12.

**FIGURE 2.12** Competition for Elected Office

		Republican candidate		
		<i>Moderate</i>	<i>Moderately conservative</i>	<i>Conservative</i>
Democratic candidate	<i>Moderate</i>	1.4	6.3	8.2
	<i>Moderately liberal</i>	3.6	5.5	9.4
	<i>Liberal</i>	2.8	4.9	7.7

The payoffs reflect these two forces: a preference to be elected with a platform closer to one's ideal, but also a desire to be elected. Note that a candidate's payoff is higher when his or her rival is more extreme, as this makes it easier to get elected. For example, if the Democratic candidate supports a moderately liberal platform, then his payoff rises from 3 to 5 to 9 as the Republican candidate's platform becomes progressively more conservative. Note also that as one goes from (*moderate, moderate*) to (*moderately liberal, moderately conservative*) to (*liberal, conservative*), each candidate's payoff rises, since, for all three strategy pairs, the candidate has an equal chance of winning (the candidates are presumed equally distant from what moderate voters want) and prefers to be elected with a platform closer to his or her own ideology.

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► SITUATION: THE SCIENCE 84 GAME

The magazine *Science 84* came up with the idea of running the following contest for its readership: Anyone could submit a request for either \$20 or \$100. If no more than 20% of the submissions requested \$100, then everybody would receive the amount he or she requested. If more than 20% of the submissions asked for \$100, then everybody would get nothing.

The set of players is the set of people who are aware of the contest. The strategy set for a player is made up of three elements: *do not send in a request*, *send in a request for \$20*, and *send in a request for \$100*. Let us suppose that each player's payoff is the amount of money received, less the cost of submitting a request, which we'll assume is \$1 (due to postage and the time it takes to write and mail a submission).

In writing down player *i*'s payoff function, let *x* denote the number of players (excluding player *i*) who chose the strategy *send in a request for \$20* and *y* denote the number of players (excluding player *i*) who chose *send in a request for \$100*. Then player *i*'s payoff function is:

$$\begin{cases} 0 & \text{if } i \text{ chooses } \textit{do not send in a request} \\ 19 & \text{if } i \text{ chooses } \textit{send in a request for \$20} \text{ and } \frac{y}{x+y+1} \leq .2 \\ 99 & \text{if } i \text{ chooses } \textit{send in a request for \$100} \text{ and } \frac{y+1}{x+y+1} \leq .2 \\ -1 & \text{if } i \text{ chooses } \textit{send in a request for \$20} \text{ and } .2 < \frac{y}{x+y+1} \\ -1 & \text{if } i \text{ chooses } \textit{send in a request for \$100} \text{ and } .2 < \frac{y+1}{x+y+1} \end{cases}$$

For example, if player *i* requested \$20, and no more than 20% of the submissions requested \$100 (i.e.,  $\frac{y}{x+y+1} \leq .2$ ), then she receives \$20 from *Science 84*, from which we need to subtract the \$1 cost of the submission.

Although it would be great to know what happened, *Science 84* never ran the contest, because Lloyd's of London, the insurer, was unwilling to provide insurance for the publisher against any losses from the contest.

## 2.6 Moving from the Extensive Form and Strategic Form

FOR EVERY EXTENSIVE FORM GAME, there is a unique strategic form representation of that game. Here, we'll go through some of the preceding examples and show how you can derive the set of players (that one's pretty easy), the strategy sets, and the payoff functions in order to get the corresponding strategic form game.

► SITUATION: BASEBALL, II

Consider the Baseball game in Figure 2.2. The strategy set of the Orioles' manager includes two elements: (1) Substitute Gibbons for Lopez and (2) retain Lopez. As written down, there is a single information set for the Yankees' manager, so his strategy is also a single action. His strategy set comprises (1) substitute Johnson for Rivera and (2) retain Rivera. To construct the payoff matrix, you just need to consider each of the four possible strategy profiles and determine to which terminal node each of them leads.

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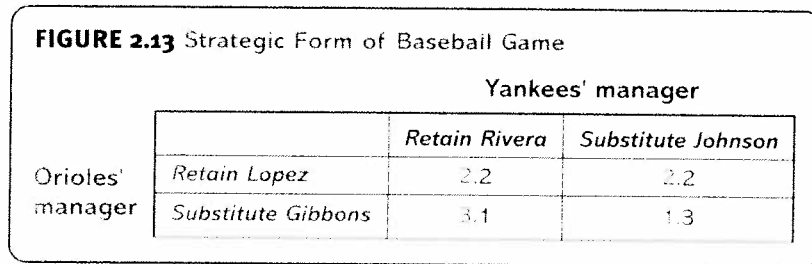
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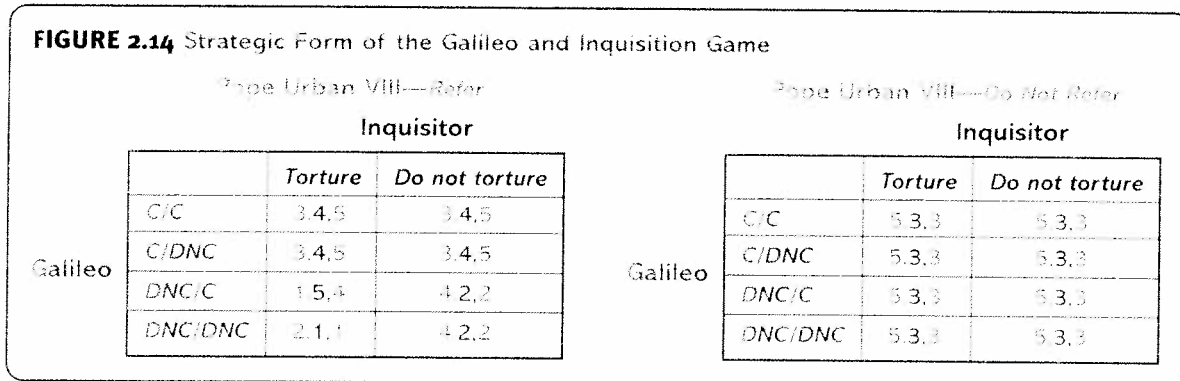
If the strategy profile is *(retain Lopez, retain Rivera)*, then the payoff is 2 for the Orioles' manager and 2 for the Yankees' manager, since Lopez bats against Rivera. The path of play, and thus the payoffs, are the same if the profile is instead *(retain Lopez, substitute Johnson)*, because *substitute Johnson* means "Put in Johnson if Gibbons substitutes for Lopez". Since the latter event doesn't occur when the Orioles' manager chooses *retain Lopez*, Johnson is not substituted. When the strategy profile is *(substitute Gibbons, retain Rivera)*, Gibbons bats against Rivera and the payoff pair is (3,1), with the first number being the payoff for the Orioles' manager. Finally, if the strategy profile is *(substitute Gibbons, substitute Johnson)*, Gibbons bats against Johnson and the payoff pair is (1,3). The payoff matrix is then as depicted in FIGURE 2.13.



► SITUATION: GALILEO GALILEI AND THE INQUISITION, II

Referring back to Figure 2.3, we see that Galileo has two information sets: one associated with Pope Urban VIII's referring the case to the Inquisitor and the other for the situation when it is referred, Galileo does not confess, and the Inquisitor tortures Galileo. A strategy for Galileo is, then, a pair of actions. We'll let *C/DNC* (*Confess/Do Not Confess*) denote the strategy for Galileo in which he confesses at the first information set and does not confess at the second. The other three strategies—*C/C*, *DNC/C*, and *DNC/DNC*—are defined analogously. The Inquisitor has one information set—when the Pope refers the case and Galileo does not confess—and two actions, all of which gives him two strategies: *torture* and *do not torture*. Urban VIII also has one information set, which is the initial node, and as he can either refer the case or not, he has two strategies: *refer* and *do not refer*.

As shown in the payoff matrix in FIGURE 2.14, Galileo chooses a row, the Inquisitor chooses a column, and the Pope chooses a matrix. The first number



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in a cell is Galileo's payoff, the second is the Inquisitor's payoff, and the third is the Pope's payoff. In filling out the matrix, consider, for example, the strategy profile (Refer, DNC/C, Do not torture). The ensuing path is that the Pope refers the case, Galileo does not confess, and the Inquisitor does not torture Galileo. The payoffs are (4,2,2), as shown in the figure. Note that if the Pope chooses not to refer the case, then the payoffs are (5,3,3), regardless of the strategies chosen by Galileo and the Inquisitor, since they don't get a chance to act. Similarly, if the Pope refers and Galileo initially confesses (by choosing either strategy C/C or strategy C/DNC), then the payoffs are the same whether the Inquisitor intends to torture or not, because Galileo's confession means that the Inquisitor doesn't get the opportunity to move.

► SITUATION: HAGGLING AT AN AUTO DEALERSHIP, II

This game is more complicated than the ones we have considered thus far. Consider the version in Figure 2.5. Marcus has four information sets: (1) the initial node, (2) the node associated with his having offered  $p^M$  and Donna's having rejected it and made a counteroffer of  $p^L$ , (3) the node associated with his having offered  $p^H$  and Donna's having rejected it and made a counteroffer of  $p^M$ , and (4) the node associated with his having offered  $p^H$  and Donna's having rejected it and made a counteroffer of  $p^L$ . Marcus's strategy template is then as follows:

- At the initial node, offer \_\_\_\_ [fill in  $p^L$ ,  $p^M$ , or  $p^H$ ].
- If I offered  $p^M$  and Donna rejected it and offered  $p^L$ , then \_\_\_\_ [fill in *accept* or *reject*].
- If I offered  $p^H$  and Donna rejected it and offered  $p^M$ , then \_\_\_\_ [fill in *accept* or *reject*].
- If I offered  $p^H$  and Donna rejected it and offered  $p^L$ , then \_\_\_\_ [fill in *accept* or *reject* and *offer  $p^M$* ].

If you write them all out, you will see that there are 24 distinct strategies for Marcus—in other words, 24 different ways in which to fill out those four blanks.

- Donna has four information sets, and her strategy template is the following:
- If Marcus offered  $p^L$ , then \_\_\_\_ [fill in *accept* or *reject*].
  - If Marcus offered  $p^M$ , then \_\_\_\_ [fill in *accept*, *reject* and *offer  $p^L$* , or *reject* and *leave*].
  - If Marcus offered  $p^H$ , then \_\_\_\_ [fill in *accept*, *reject* and *offer  $p^L$* , *reject* and *offer  $p^M$* , or *reject* and *leave*].
  - If Marcus offered  $p^H$ , I rejected and offered  $p^L$ , and Marcus rejected and offered  $p^M$ , then \_\_\_\_ [fill in *accept* or *reject*].

Donna has 48 strategies available to her. These are a lot of strategies, but keep in mind the complete nature of a strategy: no matter where Donna finds herself in the game, her strategy tells her what to do.

- Suppose Marcus and Donna chose the following pair of strategies. For Marcus:
- At the initial node, offer  $p^H$ .
  - If I offered  $p^M$  and Donna rejected it and offered  $p^L$ , then *reject*.
  - If I offered  $p^H$  and Donna rejected it and offered  $p^M$ , then *accept*.

If I offered  $p^H$  and Donna rejected it and offered  $p^L$ , then *reject and offer  $p^M$* .

For Donna:

If Marcus offered  $p^L$ , then *accept*.

If Marcus offered  $p^M$ , then *accept*.

If Marcus offered  $p^H$ , then *reject and offer  $p^L$* .

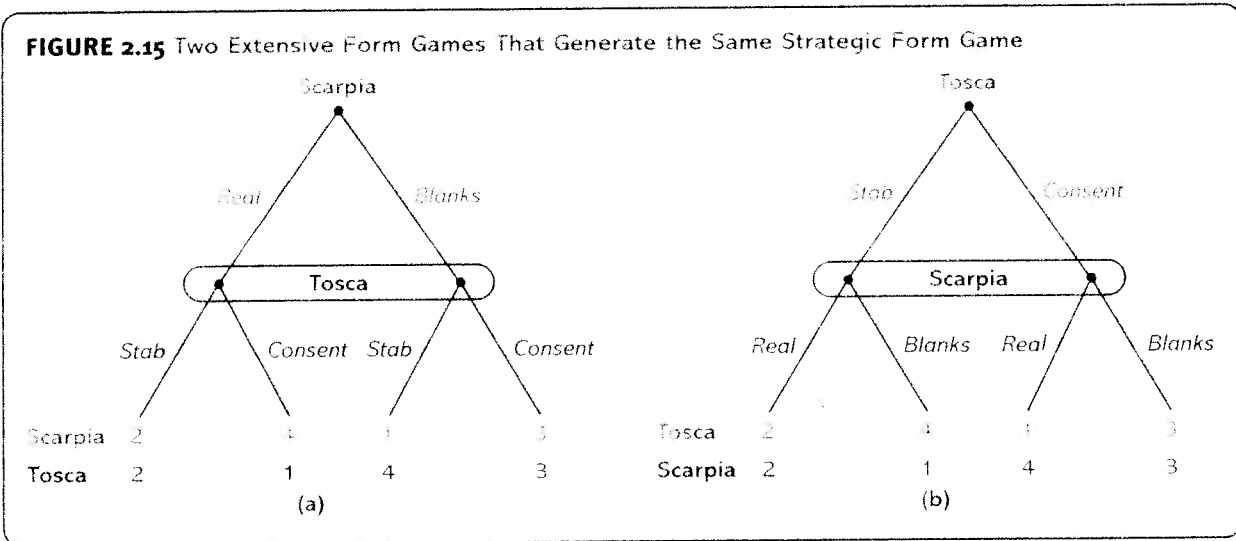
If Marcus offered  $p^H$ , I rejected and offered  $p^L$ , and Marcus rejected and offered  $p^M$ , then *accept*.

With this strategy pair, let us determine the sequence of play that logically follows and thereby the associated payoffs. At the initial node, Marcus offers a price of  $p^H$ , as prescribed by his strategy. According to Donna's strategy, she rejects the offer and counters with a price of  $p^L$ . In response to that offer, Marcus's strategy tells him to reject it and counteroffer with  $p^M$  (reading from the bottom line of his strategy). Finally, Donna's strategy has her accept the offer of  $p^M$ . The path of play that emerges is then as follows: Marcus offers a price of  $p^H$ , Donna rejects the offer and proposes a price of  $p^L$ , Marcus rejects and counters with a price of  $p^M$ , and Donna accepts. The transaction is then made at a price of  $p^M$ . For this strategy pair, the associated payoffs are  $(p^M - p^M)$  or zero, for Donna and  $2(p^M - p^L)$ , for Marcus.

### 2.7 Going from the Strategic Form to the Extensive Form

ALTHOUGH EVERY EXTENSIVE FORM GAME has a unique strategic form game associated with it, the same strategic form game can be associated with more than one extensive form game. This means that when we move from the extensive form to the strategic form, we lose some information, but, as we'll explain, the lost information is irrelevant.

Shown in **FIGURE 2.15** are two extensive form games, both of which generate the strategic form game in Figure 2.11. In the game in Figure 2.15(a),



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Scarpia moves first and then Tosca moves, but Tosca has only one information set, which indicates that she doesn't know what Scarpia chose when she decides between *stab* and *consent*. By this time, it ought to be straightforward to show that this extensive form game produces the strategic form game depicted in Figure 2.11.

The game in Figure 2.15(b) is the same as that in Figure 2.15(a), except that the sequencing of players has been reversed: still, it produces the same strategic form game, and it makes sense that it does. We've previously argued that what matters is not the chronological order of moves, but rather what players know when they act. In both of these extensive form games, Scarpia doesn't know Tosca's move when he acts; in the game in Figure 2.15(a), it is because he moves first, and in the game in Figure 2.15(b), it is because his information set includes both of Tosca's actions. Similarly, in both games, Tosca doesn't know what Scarpia has told the firing squad when she makes her choice.

#### 2.4 CHECK YOUR UNDERSTANDING

Write down the strategic form game for the Mugging game in Figure 2.8.

## 2.8 Common Knowledge

JACK AND KATE ARE TO meet at the French restaurant Per Se in New York City. Jack has since learned that the restaurant is closed today, so he e-mails Kate, suggesting that they meet at 7 P.M. at Artie's Delicatessen, their second-favorite place. Kate receives the e-mail on her BlackBerry and e-mails back to Jack, saying she'll be there. Jack receives her confirmation. Kate shows up at Artie's at 7 P.M. and Jack is not there. She wonders whether Jack received her reply. If he didn't, then he might not be sure that she had received the message, and thus he may have gone to Per Se with the anticipation that she would go there. It's 7:15, and Jack is still not there, so Kate leaves to go to Per Se. It turns out that Jack was just delayed, and he's surprised to find that Kate is not at Artie's when he arrives there.

The problem faced by Jack and Kate is what game theorists call a lack of *common knowledge*. Jack knows that Per Se is closed. Kate knows that Per Se is closed, because she received Jack's message telling her that. Jack knows that Kate knows it, since he received Kate's confirmation, and obviously, Kate knows that Jack knows it. But Kate doesn't know that Jack knows that Kate knows that Per Se is closed, because Kate isn't sure that Jack received her confirming message. The point is that it need not be enough for Jack and Kate to both know that the restaurant is closed: they may also need to know what the other knows.

To be a bit more formal here, I'm going to define what it means for an event (or a piece of information) to be common knowledge. Let  $E$  denote this event. In the preceding example,  $E$  is "Per Se is closed and meet at Artie's."  $E$  is **common knowledge** to players 1 and 2 if

- 1 knows  $E$  and 2 knows  $E$ .
- 1 knows that 2 knows  $E$  and 2 knows that 1 knows  $E$ .
- 1 knows that 2 knows that 1 knows  $E$  and 2 knows that 1 knows that 2 knows  $E$ .
- 1 knows that 2 knows that 1 knows that 2 knows  $E$  and 2 knows that 1 knows that 2 knows that 1 knows  $E$ .
- And so on, and so on.

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Are we there yet? No, because this goes on *ad infinitum*. Common knowledge is like the infinite number of reflections produced by two mirrors facing each other. Here, the “reflection” is what a player knows. Common knowledge, then, is much more than players knowing something: it involves them knowing what the others know, and knowing what the others know about what the others know, and so forth.

The concept of common knowledge is quite crucial because an underlying assumption of most of what we do in this book is that the game is common knowledge to the players. Each player knows that the game that is being played, each knows that the others know that the game that is being played, and so on.

Of course, here we are talking about an abstraction, for is anything ever truly common knowledge? Even if we’re sitting beside each other watching television, and a weather bulletin flashes along the bottom of the screen, am I sure that you saw it? Probably. But can I be sure that *you* saw *me* watching the bulletin? Possibly. But can I be sure that you saw *me watching you watching the bulletin*? Perhaps not. Although, in reality, knowledge may not hold for the entire infinite levels of beliefs required to satisfy common knowledge, it may not be a bad approximation for a lot of settings, in the sense that people act “as if” something is common knowledge.

Before we move on, let me describe an intriguing puzzle that conveys the power of common knowledge. A group of recently deceased individuals stand before the pearly gates to heaven. St. Peter is waiting for them there. (Bear with me if you don’t buy into the whole St. Peter shtick.) He tells them that only saints may enter right away, and a saint is demarcated with a halo over the head. Those who are not saints, but who try to enter, will be banished to hell. Those who are not saints and do not try to enter will go to purgatory for a while and then enter heaven. There is one problem in determining whether you are a saint: No one sees whether there is a halo over his or her own head, though each sees the halos over the heads of others. St. Peter provides one last piece of information: he announces that there is at least one saint in the group.

St. Peter begins by inviting anyone to walk through the gates. If no one does, he then asks again, and so forth. Will the saints be able to figure out who they are? They can if it is common knowledge that there is at least one saint among them.

To see how the argument operates, suppose that there is, in fact, exactly one saint. This is not initially known; everyone knows only that there is *at least one* saint. The person who is that singular saint will look around and see no halos over the heads of the others. Since he knows that there is at least one saint, he concludes that he must have a halo over his head. He then enters heaven. Admittedly, that was easy and, in fact, didn’t require common knowledge, but only knowing that there is at least one saint.

Now suppose there are instead two saints in the group, and let’s name them Tyrone and Rita. During the first calling by St. Peter, each person looks around and sees at least one person with a halo. (Those who are not saints will see two people with halos.) No one can conclude that he (or she) is a saint, so no one walks through the gates. Let us remember that each person knows that the others know that there is at least one saint. That no one walked through the gates must mean that each person must have seen at least one other person with a halo, for if the person hadn’t, then she could infer that she is a saint.

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Since everyone, including a person with a halo, saw at least one person with a halo, there must be at least two saints. Since Tyrone sees exactly one other person with a halo (Rita), and he knows that there are at least two saints, Tyrone concludes he, too, must have a halo. By an analogous logic, Rita draws the conclusion that she has a halo. They both walk through the gates on the second calling.

Okay, we could solve the problem when there are two saints. But can we do it when there are three saints? Most definitely, and what allows us to do so is that each person knows that there is at least one saint, each person knows that everyone else knows that there is at least one saint, and each person knows that everyone else knows that everyone else knows that there is at least one saint. Do you dare follow me down this daunting path of logic?

Suppose the group has three saints, and their names are Joan, Miguel, and Tamyra. As in the case when there are two saints, no one can initially conclude that he or she is a saint. Because each person sees at least two halos (Joan, Miguel, and Tamyra each see two, and everyone else sees three), knowing that there is at least one saint doesn't tell you whether you have a halo. So no one enters during the first calling. Since no one entered then, as we argued before, everyone infers that everyone must have seen a halo, which means that there must be at least two saints. So what happens in the second calling? Since there are in fact three saints, everyone sees at least two halos during the second calling, in which case no one can yet conclude that he or she is a saint. Now, what can people infer from the absence of anyone entering heaven during the second calling? Everyone concludes that everyone, *including those folks who have halos*, must have seen at least two halos. Hence, there must be at least three saints. Since Joan sees only two halos—those above the heads of Miguel and Tamyra—Joan must have a halo. Thus, she walks through the gates on the third calling, as do Miguel and Tamyra, who deploy the same logic. The three saints figure out who they are by the third calling.

Suppose there are  $n$  saints? The same argument works to show that no one enters until the  $n$ th calling by St. Peter, at which time all  $n$  saints learn who they are and enter. To derive that conclusion, it takes  $n$  levels of knowledge: everyone knows that there is at least one saint, everyone knows that everyone knows that there is at least one saint, everyone knows that everyone knows that everyone knows that there is at least one saint, and so on, until the  $n$ th level. Hence, if it is common knowledge that there is at least one saint, then the saints can always eventually figure out who they are.

It is said that, in order to succeed, it's not what you know, but whom you know. However, game theorists would say that it's not what you know, but what you know about what others know, and what you know about what others know about what you know, and . . .

## 2.9 A Few More Issues in Modeling Games

### ■ Can a player forget?

In the movie *Memento*, the character Leonard Shelby suffers from an inability to create long-term memories due to psychological trauma associated with his wife's murder. In spite of this handicap, he is in pursuit of the killer. Some of the things he learns he writes down so that he'll have the information

later, and the really important stuff he learns he has tattooed on himself. Whatever information is not written down will be forgotten. Leonard is thus cognizant of his memory deficiency and, in fact, uses it strategically. At one point, he writes down the name of someone as the murderer, even though he knows that that person is not the murderer. But he also knows that he'll read it in a few minutes and believe what his note says, because he will have forgotten that it is not true. In this way, he is committing himself to soon believing that he's found the killer, which will allow him to experience the satisfaction of vengeance when he murders him.

In the parlance of game theory, Leonard Shelby has **imperfect recall**, in that he does not necessarily know things that he previously knew. Although game theory can allow for that possibility, our attention in this book will be limited to the case of **perfect recall**, so a player who knows something at one point in the game will know it at any later stage of the game. For example, in the Iraq WMD game, when Iraq decides whether to refuse inspections (when requested by the United Nations), it remembers whether it had WMD (a move made at the start of the game). Similarly, in the game involving haggling at the auto dealership, if Marcus initially made an offer of  $p^H$ , and Donna refused it and counteroffered with  $p^L$ , Marcus remembers that he had originally offered  $p^H$  when he decides whether to accept Donna's offer of  $p^L$  or reject it and counteroffer with  $p^M$ . A situation in which you might want to allow for imperfect recall is the game of Concentration discussed in Chapter 1, since, in fact, the imperfect memories of players constitute an essential feature of the game. Alas, that is not a matter we will take on in this book.

### ■ Can a player change the game?

Consider the kidnapping situation in the movie *Ransom*. Mel Gibson plays the character Tom Mullen, whose son, Sean, is kidnapped by a corrupt police officer named Jimmy Shaker. Initially, the situation operates like a standard kidnapping. Shaker demands a 2 million dollar ransom. While going to make the ransom drop, Tom Mullen becomes convinced that the kidnappers have no intention of releasing his son. He then decides to go to the local television station and is filmed live making the following announcement:

*The whole world now knows my son, Sean Mullen, was kidnapped, for ransom, three days ago. This is a recent photograph of him. Sean, if you're watching, we love you. And this is what waits for the man that took him. This is your ransom. Two million dollars in unmarked bills, just like you wanted. But this is as close as you'll ever get to it. You'll never see one dollar of this money, because no ransom will ever be paid for my son. Not one dime, not one penny. Instead, I'm offering this money as a reward on your head. Dead or alive, it doesn't matter. So congratulations, you've just become a 2 million dollar lottery ticket, except the odds are much, much better. Do you know anyone that wouldn't turn you in for 2 million dollars? I don't think you do. But, this is your last chance, if you return my son, alive, uninjured, I'll withdraw the bounty. With any luck, you can simply disappear. Understand. You will never see this money. Not one dollar. So you still have a chance to do the right thing. If you don't, well, then, God be with you, because nobody else on this Earth will be.*

It is clear that Tom Mullen had a "brainstorm" of converting the ransom into a bounty. Furthermore, it is natural to suppose that this possibility was

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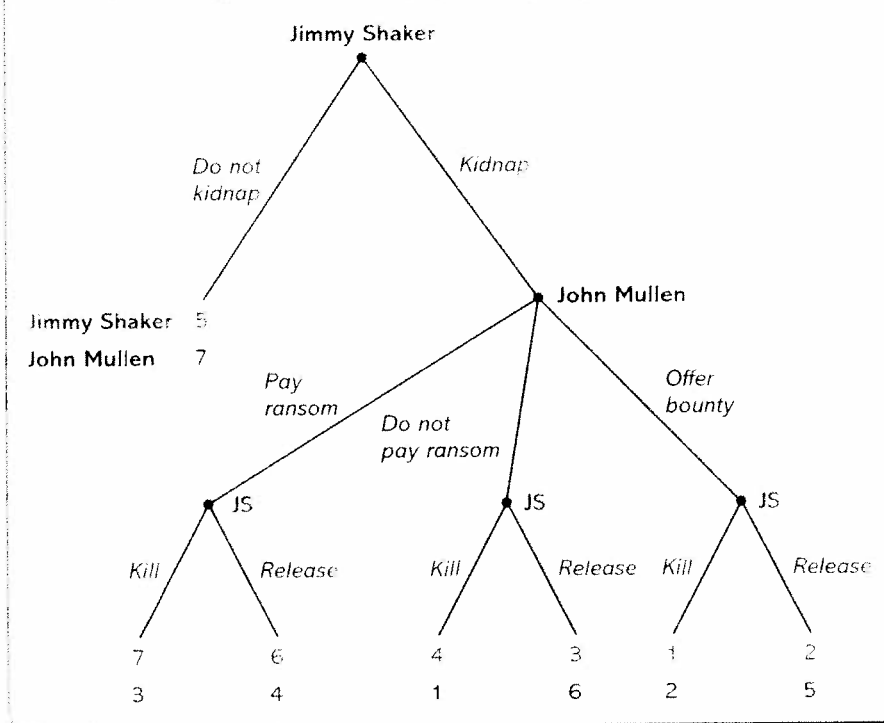
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not one that Jimmy Shaker had considered. In a way, the players started with the game looking like that in Figure 2.1, but then Tom Mullen “changed” the game to something else. Unfortunately, game theory does not allow for such changes or innovations. A key assumption of game theory is that the game is initially understood and agreed upon by the players; the rules of the game are common knowledge.

What we can do, however, is modify the game to that in FIGURE 2.16. John Mullen now has the option of paying the ransom, offering a bounty, or doing nothing, in response to each of which Jimmy Shaker has the two options of releasing or killing Sean Mullen. This game is understood by the players when they start it and thus does not allow for the possibility of Tom Mullen “surprising” Jimmy Shaker by offering a bounty. True innovation is not a feature that current game theory can encompass. Thus, the answer is that players are *not* allowed to change the game. However, we can always enrich the game and give players more options.

**FIGURE 2.16** Extensive Form for the Film *Ransom*



■ **Does the game have to be factually accurate?**

If our objective in formulating and then solving a game is to understand behavior, then what matters is not what is factually or objectively true, but rather what is perceived by the players. Their behavior will be driven by their preferences and what they believe, whether those beliefs are in contradiction with reality or not. Thus, a game ought to represent players' environment *as it is perceived by them*.

If a player is a member of a Native American tribe in the 19th century which believes that a tribal leader has magical powers, we need to recognize that belief—regardless of whether or not it is true—if we are to understand their behavior. Or if a player in the 21st century believes that flying a plane into a building can improve his well-being in the afterlife, then we need similarly to recognize that belief, no matter how wrong or misguided it may be.

### Summary

When the French Impressionist painter Claude Monet viewed a London building, a French cathedral, or a lily pond in his backyard, he painted, not reality, but his impression of it. Modeling real-life encounters between people is similarly an art form, though admittedly not one worth framing and hanging on your wall. Real life is complicated, nuanced, and messy, and a social scientist who wants to understand it must distill its essential features if he is to construct a simple and parsimonious model. Doing so requires creativity, insight, and judgment. While game theory cannot bring those attributes to the table, it can provide the tools for the intelligent observer who has such traits to build a model that will shed light on why people do the things they do.

In this chapter, we have reviewed the two frameworks for constructing a game-theoretic model of a strategic situation. An **extensive form game** uses a tree structure to depict the sequence in which players make decisions and describes the circumstances surrounding those decisions, including the actions available to a player and what the player knows regarding what has happened in the game. That knowledge is represented by an **information set** which encompasses all those paths in the game that a player is incapable of distinguishing among. The concept of an information set allows us to model the many different contexts in which decisions are made while we lack relevant facts. An information set can embody a situation of **perfect information**, in which a player knows all that has transpired thus far in the game, or one of **imperfect information**, in which a player has some uncertainty in regard to what other players have done. Key to describing behavior is knowing what players care about, so an extensive form game also describes the well-being, or **payoff**, that a player assigns to an outcome of the game.

A **strategic form game** has a more concise format than the extensive form game has. A strategic form game is defined by the **set of players**, the **strategy set** of each player, and the **payoff function** of each player. A player's decision making involves the selection of a strategy from his or her strategy set, where a **strategy** is a fully specified decision rule for how to play a game. A payoff function tells us how a player evaluates any collection of strategies (one for each of the players in the game).

As will be revealed in the ensuing chapters, crucial to both predicting behavior and prescribing behavior is knowing what each player knows about the other players, and this knowledge includes what each player believes about the other players. A central underlying assumption is that the game is **common knowledge** to the players. This means not only that players agree

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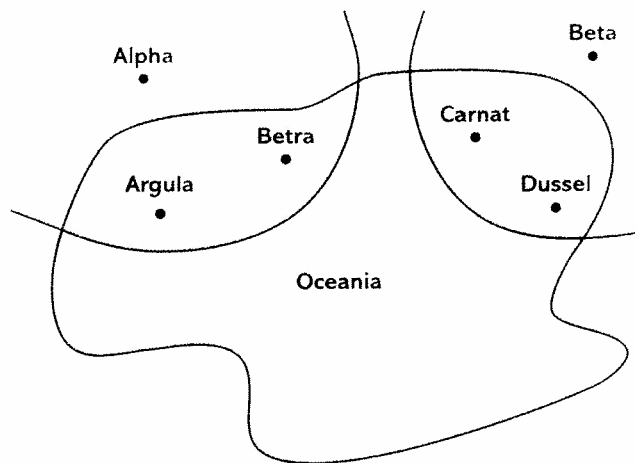


on the game that is being played, but also that each player knows what the other players believe about the game, and so forth. Common knowledge is like the perfect circle; it is a concept that does not exist in reality, but nevertheless is a useful abstraction for understanding the world within which we live.

## EXERCISES

1. The countries of Oceania and Eurasia are at war.<sup>5</sup> As depicted in **FIGURE PR2.1**, Oceania has four cities—Argula, Betra, Carnat, and Dussel—and it is concerned that one of them is to be bombed by Eurasia. The bombers could come from either base Alpha, which can reach the cities of Argula and Betra, or base Beta, which can reach either Carnat or Dussel. Eurasia decides which one of these four cities to attack. Oceania doesn't know which one has been selected, but does observe the base from which the bombers are flying. After making that observation, Oceania decides which one (and only one) of its four cities to evacuate. Assign a payoff of 2 to Oceania if it succeeds in evacuating the city that is to be bombed and a payoff of 1 otherwise. Assign Eurasia a payoff of 1 if the city it bombs was not evacuated and a zero payoff otherwise. Write down the extensive form game.

**FIGURE PR2.1**



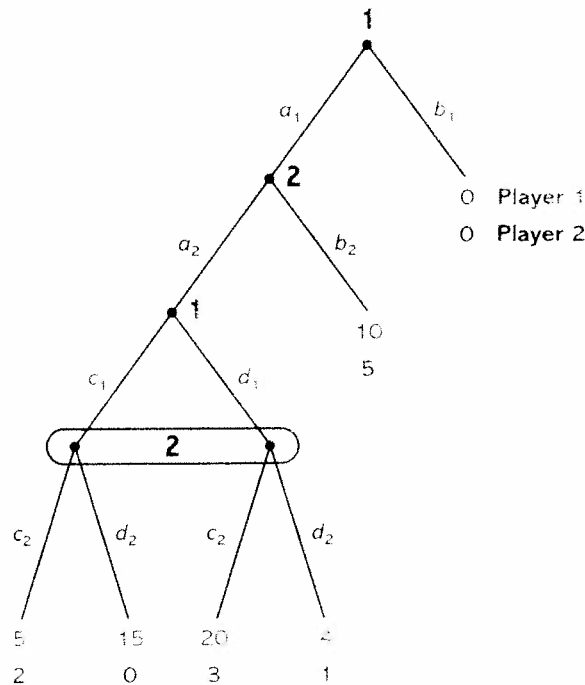
2. Player 1 moves initially by choosing among four actions:  $a$ ,  $b$ ,  $c$ , and  $d$ . If player 1 chose anything but  $d$ , then player 2 chooses between  $x$  and  $y$ . Player 2 gets to observe the choice of player 1. If player 1 chose  $d$ , then player 3 moves by choosing between  $left$  and  $right$ . Write down the extensive form of this setting. (You can ignore payoffs.)

3. Consider a setting in which player 1 moves first by choosing among three actions:  $a$ ,  $b$ , and  $c$ . After observing the choice of player 1, player 2 chooses among two actions:  $x$  and  $y$ . Consider the following three variants as to what player 3 can do and what she knows when she moves:
  - a. If player 1 chose  $a$ , then player 3 selects among two actions: *high* and *low*. Player 3 knows player 2's choice when she moves. Write down the extensive form of this setting. (You can ignore payoffs.)
  - b. If player 1 chose  $a$ , then player 3 selects among two actions: *high* and *low*. Player 3 does not know player 2's choice when she moves. Write down the extensive form of this setting. (You can ignore payoffs.)
  - c. If player 1 chose either  $a$  or  $b$ , then player 3 selects among two actions: *high* and *low*. Player 3 observes the choice of player 2, but not that of player 1. Write down the extensive form of this setting. (You can ignore payoffs.)
4. Return to the game involving the U.S. Court of Appeals in Section 2.2.
  - a. Suppose, at the start of the game, it is known by all that Judge Z will read only the brief of Ms. Hasenpfeffer. Write down the corresponding extensive form game. You may exclude payoffs.
  - b. Suppose, at the start of the game, it was known by all that Judge X would vote first and reveal his vote to Judges Y and Z before they vote simultaneously. Write down the corresponding extensive form game. You may exclude payoffs.
5. The city council is to decide on a proposal to raise property taxes. Suppose Ms. Tuttle is the chair and the Council's other two members are Mr. Jones and Mrs. Doubtfire. The voting procedure works as follows: Excluding the chair, Mr. Jones and Mrs. Doubtfire simultaneously write down their votes on slips of paper. Each writes either *for* or *against* the tax increase. The secretary of the city council then opens the slips of paper and announces the vote tally. If the secretary reports that both slips say *for*, then the tax increase is implemented and the game is over. If both vote *against*, then the tax increase is not implemented and, again, the game is over. However, if it is reported that the vote is one *for* and one *against*, then Ms. Tuttle has to vote. If she votes *for*, then the tax increase is implemented, and if she votes *against*, then it is not. In both cases, the game is then over. As to payoffs, if the tax increase is implemented, then Mrs. Doubtfire and Mr. Jones each receive a payoff of 3. If the tax increase proposal fails, then Mrs. Doubtfire has a payoff of 4 and Mr. Jones's payoff is 1. As for Ms. Tuttle, she prefers to have a tax increase—believing that it will provide the funds to improve the city's schools—but would prefer not to be on record as voting for higher taxes. Her payoff from a tax increase when her vote is not required is 5, her payoff from a tax increase when her *for* vote is required is 2, and her payoff from taxes not being increased is zero (regardless of whether or not she voted). Write down the extensive form of the game composed of Ms. Tuttle, Mr. Jones, and Mrs. Doubtfire.
6. Consider a contestant on the legendary game show *Let's Make a Deal*. There are three doors, and behind two doors is a booby prize (i.e., a prize of little value), while behind one door is a prize of considerable value, such as an automobile. The doors are labeled 1, 2, and 3. The strategic situation starts when, prior to the show, host Monty Hall selects one of the three doors behind which to place the good prize. Then, during the show, a
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contestant selects one of the three doors. After its selection, Monty opens up one of the two doors not selected by the contestant. In opening up a door, a rule of the show is that Monty is prohibited from opening the door with the good prize. After Monty Hall opens a door, the contestant is then given the opportunity to continue with the door originally selected or switch to the other unopened door. After the contestant's decision, the remaining two doors are opened.

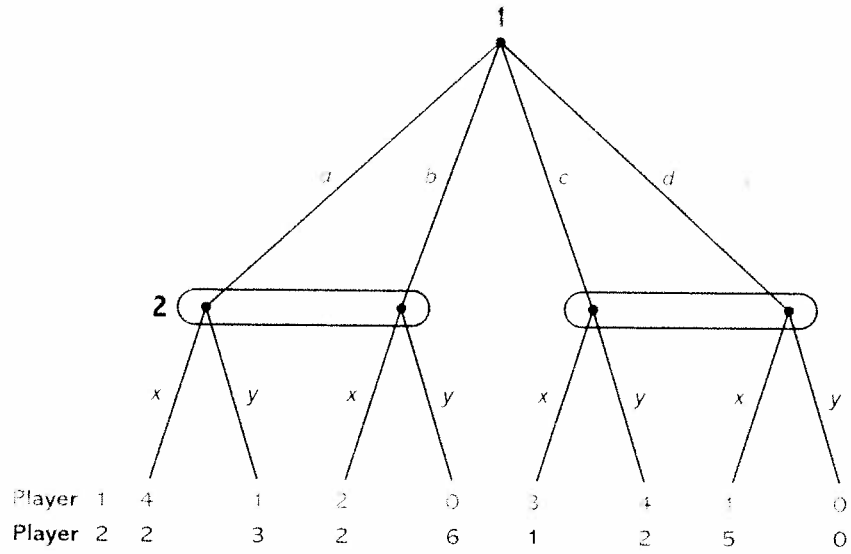
- a. Write down an extensive form game of *Let's Make a Deal* up to (but not including) the stage at which the contestant decides whether to maintain his original choice or switch to the other unopened door. Thus, you are to write down the extensive form for when (1) Monty Hall chooses the door behind which the good prize is placed, (2) the contestant chooses a door, and (3) Monty Hall chooses a door to open. You may exclude payoffs.
  - b. For the stage at which the contestant decides whether or not to switch, write down the contestant's collection of information sets. In doing so, denote a node by a triple, such as 3/2/1, which describes the sequence of play leading up to that node. 3/2/1 would mean that Monty Hall put the good prize behind door 3, the contestant initially selected door 2, and Monty Hall opened door 1.
7. For the Iraq War game in Figure 2.10, write down the strategy sets for the three players.
  8. For the extensive form game in **FIGURE PR2.8**, derive its corresponding strategic form.

**FIGURE PR2.8**



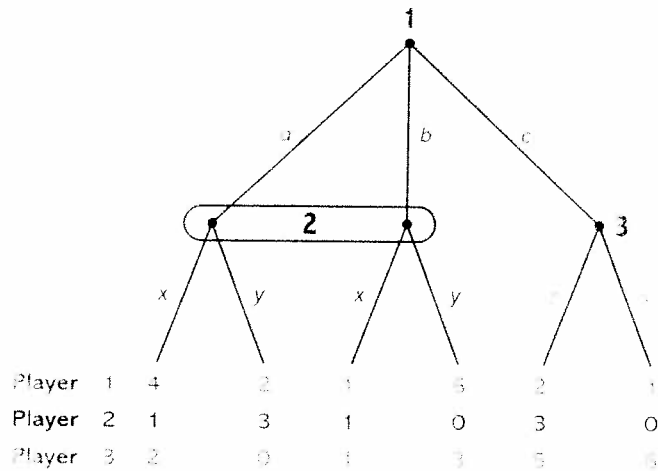
9. Write down the strategic form game for the extensive form game in FIGURE PR2.9.

**FIGURE PR2.9**



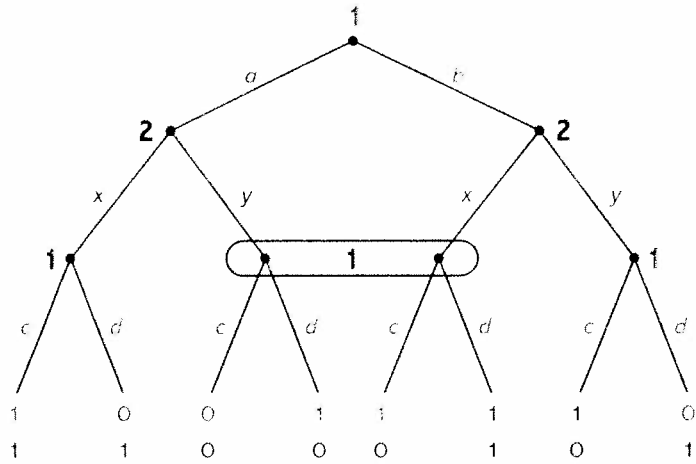
10. Write down the strategic form game for the extensive form game in FIGURE PR2.10.

**FIGURE PR2.10**

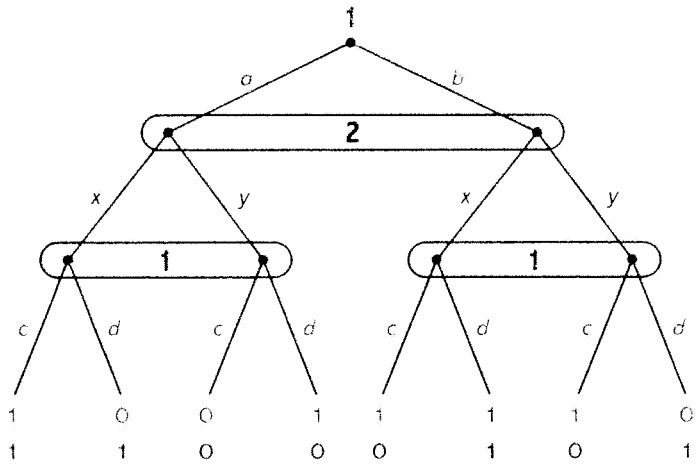


11. Three extensive form games are shown in FIGURE PR2.11. State which of them, if any, violate the assumption of perfect recall. Explain your answer.

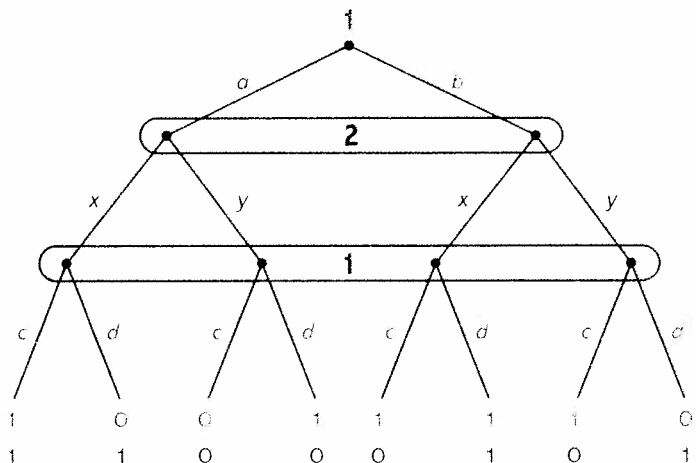
FIGURE PR2.11



(a)



(b)



(c)

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answer:

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1. John Thorn and Pete Palmer, *The Hidden Game of Baseball* (New York: Doubleday/Dolphin, 1985). The seasons analyzed were 1974-77.
2. Sun Tzu, *The Art of War*, translated by Thomas Cleary, Shambhala, 1998. For an examination of Sun Tzu's writings using the lens of game theory, see Emerson M. S. Niou and Peter C. Ordeshook, "A Game-Theoretic Interpretation of Sun Tzu's *The Art of War*," *Journal of Peace Research*, 31 (1994), 161-74.
3. J. D. Williams, *The Compleat Strategyst* (New York: McGraw Hill, 1954), p. 16.
4. This application of game theory was first suggested by Anatol Rapoport, "The Use and Misuse of Game Theory," *Scientific American*, 207(6): 108-18.
5. These fictitious countries appear in the George Orwell novel *1984*. If dystopia is not your bag, then you can use Freedonia and Sylvania in their place, which are fictional nations from the 1933 Marx Brothers' movie *Duck Soup*.

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