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Bertrand versus Cournot revisited*

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Summary. Bertrand criticized Cournot's analysis of the competitive process, arguing that firms should be seen as playing a strategy of setting price below competitors' prices (henceforth, the *Bertrand strategy*) instead of a strategy of accepting the price needed to sell an optimal quantity (the *Cournot strategy*). We characterize Nash equilibria in a generalized model in which firms choose among Cournot *and* Bertrand strategies. Best responses always exist in this model. For the duopoly case, we show that iterated best responses converge under mild assumptions on initial states either to Cournot equilibrium or to an equilibrium in which only one firm plays the Bertrand strategy with price equal to marginal cost and that firm has zero sales.

JEL Classification Numbers: B13, C72, D43, L13.

In Cournot's model of oligopolistic competition, each firm maximizes profits by producing an optimal quantity and then adjusting price to whatever level is needed to sell that quantity (henceforth, by playing the *Cournot strategy*).[1] Bertrand later criticized Cournot's analysis, arguing that each firm should instead be assumed to maximize profits by setting a price that undercuts competitors' prices when competitors' prices exceed cost (henceforth, by

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[1] Cournot's text makes clear that this was the strategy he considered. He stated, for instance, that a proprietor will set sales to an optimal level "by properly adjusting his price." Thus although it is now common to view firms in the Cournot model as choosing quantities and "the market" as determining price, it seems that Cournot intended his model to be of price-setting as well as quantity-setting. Indeed, it is precisely Cournot's idea that firms *indirectly* set prices that led to Bertrand's critique; Bertrand found a different answer by asking what happens if firms set prices *directly*.

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playing the *Bertrand strategy*).[2] We reevaluate Bertrand's critique, allowing *both* types of strategies to compete in the market and possibly to coexist in market equilibrium.

At issue is how the competitive process works. Cournot and Bertrand focused on different details of the competitive process, and were led to specify different mechanisms by which individual consumers' demands are allocated among competing firms: Cournot assumed that the market allocates sales equal to what any given firm produces but at a price determined by what the market will bear, while Bertrand assumed that the firm with the lowest price (if there is only one such firm) is allocated all sales. Thus the first step in the reevaluation is to specify a demand-allocating mechanism for an oligopolistic market when strategy sets are generalized to allow firms to choose among Cournot *and* Bertrand strategies (sections I).[3] The mechanism we specify has the property that it degenerates into Cournot's or Bertrand's mechanism if strategy sets are restricted to contain only Cournot or Bertrand strategies (section II).

With generalized strategy sets, we study a market with two types of firms: some set prices; and others take prices as given and set quantities. Firms tend to stay of the same type: a firm switches type if and only if it gains strictly greater profits from switching. The demand-allocating mechanism we specify then has a "leader-follower" character in that price-setting firms lead by determining endogenously the market price, quantity-setting firms act as price takers at that market price, and price-setting firms end up with sales equal to market demand at that price minus the total quantity sold by quantity-setting firms. After characterizing Nash equilibria for this generalized model (section III), we study iterated best responses or "Cournot dynamics" (section IV). For the duopoly case, we show that iterated best responses converge under mild assumptions about initial states either to

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[2] Bertrand's text makes clear that this was the strategy he considered: "whatever jointly determined price were adopted, if only one of the competitors lowers his, he gains ... all of the sales, and he will double [in the duopoly case] his returns ..."

[3] We maintain Cournot's and Bertrand's assumptions of a frictionless market for a homogeneous commodity. Given these assumptions, each firm's demand is *endogenously* determined under Cournot's and Bertrand's demand-allocating mechanisms from market demand, which is a sufficient proxy for optimization by each consumer. Under Bertrand's mechanism (and under the generalized mechanism below), each firm's demand is discontinuous in price at the lowest of the prices set by competitors; this discontinuity is the mathematical essence of Bertrand's critique. To allow for the possibility of such discontinuities and thus to allow for Bertrand's critique in studying how the competitive process might work with heterogeneous goods, it would be necessary to derive each firm's demand *endogenously* by first specifying details of the competitive process that determines which firms sell to which optimizing consumers and in which quantities when some firms set prices and some set quantities, and then aggregating over consumers. Determining firm demands endogenously in this way would be more complex than assuming that the demand faced by a firm offering a heterogeneous good is a simple (continuous), *exogenously* given function, as in, e.g., Singh and Vives (1984), Klemperer and Meyer (1986), and Jehiel and Walliser (1995).

Cournot equilibrium or to an equilibrium in which one firm plays the Bertrand strategy with price equal to marginal cost and one firm plays a Cournot strategy and receives all sales in the market.[4] The latter equilibria may be interpreted as equilibria with "potential competition."

1 An oligopoly game with generalized strategies

Consider oligopoly in a market in which $n \geq 2$ identical firms sell a homogeneous commodity that is produced without capacity constraints or fixed costs but at a constant marginal cost of v per unit; we refer to v as simply *cost*. The market is assumed frictionless in the sense that buyers never purchase from a firm if there is another firm that would sell at a lower price. To generalize Cournot's and Bertrand's analyses to allow each firm to choose among Bertrand and Cournot strategies, we first define strategies and strategy sets, and then specify a demand-allocating mechanism that determines how the vector of profits of all firms depends on the vector of strategies of all firms.

Denote firm i's strategy by σ^i and the vector of strategies played by all firms in the market by $\sigma = (\sigma^1, \dots, \sigma^n)$. Also denote the set of all strategies of firm i by Σ^i and the set of all strategies of all firms by the Cartesian product $\Sigma = \otimes_{i=1}^n \Sigma^i$. In detail, we write $\sigma^i = (p^i, q^i)$ and use the null symbol ϕ to identify the strategy that is *not* being played, assuming that either $p^i = \phi$ or $q^i = \phi$ but not both. We say *firm i plays the Bertrand strategy with price* p^i to mean that i plays $\sigma^i = (p^i, \phi)$, setting price equal to $p^i \geq 0$ and then selling as much as possible at p^i. A story that captures the idea of the Bertrand strategy is that firm *i publishes a price* at the dawn of a market day under which i commits to sell at price p^i all that customers demand during the market day; i is then treated as able to produce exactly the quantity customers demand at cost v per unit. Given any strategy vector σ, the set of all firms using Bertrand strategies under σ is $B_\sigma = \{i | \sigma^i = (p^i, \phi)\}$. Similarly, *firm i plays the Cournot strategy with quantity* q^i means that i plays $\sigma^i = (\phi, q^i)$, producing quantity $q^i \geq 0$ and then adjusting the price it charges to the maximum price at which q^i can be sold. A story in this case is that firm *i produces a quantity* q^i at the dawn of a market day and commits to sell this entire quantity during the market day at the greatest price the market will

[4] Kreps and Scheinkman (1983) provide a related analysis that might be taken as favoring Cournot equilibrium over Bertrand equilibrium. They interpret Cournot and Bertrand strategies essentially as we do. They study a two-stage game with capacity constraints in which capacity is chosen in the first stage and Bertrand-like competition occurs in the second stage; they show that the unique equilibrium under certain assumptions is Cournot equilibrium. By contrast, we study a one-stage game without capacity constraints in which competition is general in that firms choose either a Cournot strategy (produce and bring to market a given quantity) or a Bertrand strategy (set a price and possibly wait to produce until consumers place their orders); we show that iterated best responses cannot converge to an equilibrium in which a firm plays the Bertrand strategy and has positive sales.

bear. The set of all firms that play Cournot strategies under σ is $C_\sigma = \{i|\sigma^i = (\phi, q^i)\}$.[5]

Market demand is a downward sloping, twice continuously differentiable function D defined over $[0, \infty)$, where $D(p)$ is the quantity demanded by consumers at price p and $D^{-1}(Q)$ is the price that just clears a market quantity of Q. To ensure that the model has a nontrivial solution, we assume that $D^{-1}(0) > v$, that is, that the "choke price" exceeds cost. The sum of all quantities produced by firms that play Cournot strategies under strategy vector σ is $Q_\sigma = \Sigma_{i \in C_\sigma} q^i$ and the minimum of the prices charged by firms playing Bertrand strategies under σ is $p_\sigma = \min(p^i | i \in B_\sigma)$. As a convenience, we take $Q_\sigma = 0$ if no firm plays a Cournot strategy and $p_\sigma = \infty$ if no firm plays a Bertrand strategy.

We wish to specify a demand-allocating mechanism that captures the spirits of both Cournot's and Bertrand's analyses. Because the market is frictionless and there are no capacity constraints, we take all trade as occurring at the minimum of the prices charged by *all* firms; this minimum price is the *market price*. The market price and the quantities sold by each firm are determined as follows. If $D^{-1}(Q_\sigma) \leq p_\sigma$ so the price at which consumers would just purchase the total quantity produced by firms playing Cournot strategies is less than or equal to the minimum of the prices charged by firms playing Bertrand strategies, then the market price is $D^{-1}(Q_\sigma)$ and firms playing Cournot strategies supply all that buyers demand at the market price. This is the assumed outcome because the Cournot strategy is to produce an optimal quantity and then accept whatever price is needed to sell that quantity; thus when $D^{-1}(Q_\sigma) \leq p_\sigma$, each firm that plays a Cournot strategy sells its produced quantity at market price $D^{-1}(Q_\sigma)$ and all firms that play Bertrand strategies are effectively undercut by firms playing Cournot strategies and hence have zero sales. If $p_\sigma < D^{-1}(Q_\sigma)$, on the other hand, so the minimum price set by firms playing Bertrand strategies is less than the price at which consumers would just purchase the total quantity produced by firms playing Cournot strategies, then the market price is p_σ and the total quantity sold by firms playing Cournot strategies does not exhaust market demand. This is the assumed outcome because each firm that plays a Cournot strategy accepts whatever price is needed to sell the quantity the firm has produced; thus when $p_\sigma < D^{-1}(Q_\sigma)$, each firm that plays a Cournot strategy sells its produced quantity at market price p_σ, which leaves total sales of $D(p_\sigma) - Q_\sigma > 0$ to be shared among firms that play the Bertrand strategy with price p_σ.[6] When two or more firms play Bertrand strategies with the minimum price p_σ, we assume as is standard that these firms share

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[5] Similar interpretations of Bertrand and Cournot strategies are in Telser (1987, pp. 222-3).

[6] It should be stressed that the way firms playing Cournot and Bertrand strategies share market demand follows essentially from the way Cournot and Bertrand strategies are defined. The game we specify thus reflects the strategies proposed by Cournot and Bertrand (see footnotes 1 and 2). If strategies were defined differently, the game and the results (equilibria) would obviously also differ.

available sales equally. Let $B(p_\sigma)$ denote the set of firms that play the Bertrand strategy with price p_σ and let the operator # denote the number of elements in a set so $\#B(p_\sigma)$ is the number of firms that play the Bertrand strategy with price p_σ. Then if $D(p_\sigma) - Q_\sigma > 0$, sales of a firm that plays the Bertrand strategy with price p_σ are $[D(p_\sigma) - Q_\sigma]/\#B(p_\sigma)$.

Given σ, if firm i plays the Bertrand strategy $\sigma^i = (p^i, \phi)$, profits are

$$\pi^i(\sigma) = \begin{cases} (p_\sigma - v)[D(p_\sigma) - Q_\sigma]/\#B(p_\sigma) & \text{if } p^i = p_\sigma \text{ and } D(p_\sigma) \geq Q_\sigma, \\ 0 & \text{otherwise.} \end{cases} \quad (1)$$

Similarly if firm i plays the Cournot strategy $\sigma^i = (\phi, q^i)$, profits are

$$\pi^i(\sigma) = \begin{cases} (p_\sigma - v)q^i & \text{if} D(p_\sigma) \geq Q_\sigma\ , \\ \left[D^{-1}(Q_\sigma) - v\right]q^i & \text{otherwise.} \end{cases} \quad (2)$$

We use $\sigma^{-i} \in \otimes_{j \neq i} \Sigma^j$ to denote the vector of strategies played by firms other than i. The profits of firm i can then be written $\pi^i(\sigma^i, \sigma^{-i})$. We also use

$$Q_\sigma^{-i} = \begin{cases} Q_\sigma & \text{if } i \in B_\sigma \\ Q_\sigma - q^i & \text{if } i \in C_\sigma, \end{cases}$$

to denote the sum of the quantities produced by all firms that play Cournot strategies other than firm i.

The profit functions (1), (2) together with firms' strategy sets $\Sigma^1, \ldots, \Sigma^n$ constitute a game in strategic form. In Nash equilibrium, each firm maximizes its profits taking the strategies of all other firms as given. We use stars to denote equilibrium values, so Nash equilibrium is a vector $\sigma^* \in \Sigma$ such that $\pi^i(\sigma^{i*}, \sigma^{-i*}) \geq \pi^i(\sigma^i, \sigma^{-i*})$ for all $\sigma^i \in \Sigma^i$ and $i = 1, \ldots, n$.

2 Restricted strategy sets: Cournot and Bertrand equilibria

If strategy sets are restricted to contain only Cournot strategies or only Bertrand strategies, then the profit functions reduce in a way that causes the generalized model to degenerate into the Cournot model or the Bertrand model. In the first case, strategy sets are $\Sigma_C^i = \{\sigma^i \in \Sigma^i | \sigma^i = (\phi, q^i)\}$ for all $i = 1, \ldots, n$. Nash equilibrium of the game with strategy sets $\Sigma_C^1, \Sigma_C^2, \ldots, \Sigma_C^n$ is Cournot equilibrium.[7] Similarly, strategy sets restricted to contain only Bertrand strategies are $\Sigma_B^i = \{\sigma^i \in \Sigma^i | \sigma^i = (p^i, \phi)\}$ for all $i = 1, \ldots, n$. Nash equilibrium of the game with strategy sets $\Sigma_B^1, \Sigma_B^2, \ldots, \Sigma_B^n$ is Bertrand equilibrium.

[7] There may be multiple Cournot equilibria; results here do not require that equilibrium be unique. A sufficient condition for uniqueness is that $dD^{-1}(Q)/dQ + Qd^2D^{-1}(Q)/dQ^2 \leq 0$, which holds if, in addition to assumptions already made, market demand (D) is concave–see Shapiro (1989, p. 335).

3 Generalized strategy sets: Nash equilibria

The following theorem characterizes Nash equilibria of the oligopoly game with profit functions $\pi^1(\sigma), \pi^2(\sigma), \ldots, \pi^n(\sigma)$ and generalized strategy sets $\Sigma^1, \Sigma^2, \ldots, \Sigma^n$:

Theorem 1. *The Nash equilibrium set of the generalized game consists only of strategy vectors σ^* that satisfy:*

(a) *σ^* is a Cournot equilibrium; or*

(b) *$p_{\sigma^*} = v$ and either $\#B(\sigma^*) \geq 2$ and $Q_{\sigma^*} \leq D(v)$, or $\#B(\sigma^*) = 1$ and $Q_{\sigma^*} = D(v)$.*

Proof. That vectors σ^* under (a) and (b) are Nash equilibria follows because profits cannot be raised by any change of strategy. To show that no other Nash equilibria are possible, let $\sigma \in \Sigma$ satisfy neither (a) nor (b). Because Bertrand and Cournot equilibria are covered under (a) and (b), we assume without loss of generality that B_σ and C_σ are nonempty. Further, we restrict attention to cases in which $Q_\sigma \leq D(v)$, because $Q_\sigma > D(v)$ requires a market price below cost, which cannot occur in Nash equilibrium. We show that in every case, some firm has an incentive to deviate from σ so σ cannot be a Nash equilibrium. Suppose first that $p_\sigma = v$. Because σ does not satisfy (b), it must be that $Q_\sigma < D(v)$ and $\#B(p_\sigma) = 1$. This cannot occur in Nash equilibrium because the firm that plays the Bertrand strategy with price v could then raise its price slightly and still have positive sales so its profits would become positive. Now suppose that $p_\sigma > v$. If $Q_\sigma = D(v)$, then market price equals v so at least one firm in C_σ with a positive quantity can raise the price it receives by reducing its quantity slightly, thereby moving from zero to positive profits. If $Q_\sigma < D(v)$ and $\#B(p_\sigma) \geq 2$, then each firm playing p_σ has an incentive to undercut p_σ slightly. Finally if $Q_\sigma < D(v)$ and $\#B(p_\sigma) = 1$, then either $Q_\sigma \geq D(p_\sigma)$, in which case the firm that sets p_σ has no sales and hence no profits when positive profits are possible, so σ cannot be an equilibrium, or else $Q_\sigma < D(p_\sigma)$. If $Q_\sigma < D(p_\sigma)$, then the market price is p_σ and $q^i = Q_\sigma - Q_\sigma^{-i} < D(p_\sigma) - Q_\sigma^{-i}$ for a firm i playing the Cournot strategy. Such a quantity cannot be optimal for i because, from (2), i would have greater profits at a quantity equal to $D(p_\sigma) - Q_\sigma^{-i}$. □

Two categories of equilibria are covered by condition (b) of the theorem:

(i) Traditional *Bertrand equilibrium* is the case in which *all* firms play Bertrand strategies.

(ii) Bertrand-like equilibria are cases in which one or more firms play the Bertrand strategy with price equal to cost and at least one firm plays a Cournot strategy. We term these mixed equilibria "Bertrand-like" because equilibrium price equals cost, as in Bertrand equilibrium. In Bertrand-like equilibrium, firms that play the Bertrand strategy with price equal to cost act like endogenous auctioneers that set market price equal to cost, and firms that play Cournot strategies then behave as price takers. Note that there are many Bertrand-like equilibria because best responses are not single-valued if price equals cost; namely, the optimal Cournot strategy is to sell an indeterminate amount less than or equal to $D(v) - Q_{\sigma}^{-i}$.

A special case of Bertrand-like equilibria occurs when one or more firms play Bertrand strategies with price equal to cost and each of these firms has zero sales. We refer to such equilibria as *virtual Bertrand equilibria*; a virtual Bertrand equilibrium is formally a strategy vector σ^* such that $p_{\sigma^*} = v$, $\#B_{\sigma^*} > 0$, and $Q_{\sigma^*} = D(v)$. Virtual Bertrand equilibria are unrobust in a particular sense. Namely, we say that a firm is a *virtual firm* if it has zero sales in spite of playing a Bertrand strategy with price equal to the market price. Thus there is at least one virtual firm in any virtual Bertrand equilibrium. With zero sales, it is natural to imagine that such firms may cease to matter; indeed it may be reasonable to interpret firms that sell zero in equilibrium as having exited from the market. (An analogy to virtual particles that exist only briefly in the physical world would be to say that virtual firms "decay.") The unrobustness is that the strategies of the *nonvirtual* firms do not constitute a Nash equilibrium if the virtual firms are removed from the model. To express this precisely, let σ^* be a virtual Bertrand equilibrium in which m firms are nonvirtual firms and let $\tilde{\sigma}^*$ be the subvector consisting of the strategies under σ^* of the m nonvirtual firms. Because total sales of firms playing Cournot strategies satisfy $Q_{\sigma^*} = D(v)$, it follows that $Q_{\tilde{\sigma}^*} = D(v)$ and $p_{\tilde{\sigma}^*} > v$. By theorem 1, the strategy vector $\tilde{\sigma}^*$ is not a Nash equilibrium of the game with generalized strategy sets that is formed when the virtual firms under σ^* are removed from the market. An interpretation is that the virtual firms in virtual Bertrand equilibrium are like "potential competitors at the industry's doorstep" that stand ready to sell at a price equal to cost, and in so doing keep market price equal to cost.[8]

4 Generalized strategy sets: dynamics

From theorem 1, no clear *existence* argument favors Bertrand or Cournot equilibrium over the other. To evaluate whether Bertrand's critique of Cournot might imply a *dynamic* argument in favor of one equilibrium, we study Cournot's method of iterated best responses; thus dynamics here means iterated best responses from a given initial state at time $t = 0$. To define these dynamics precisely, let σ_t^i denote the strategy of firm i at time t, let σ_t^{-i} denote the vector of strategies of all firms other than i at time t, and let σ_t denote the vector of strategies of all firms at time t. Then under iterated best responses, firm $i = 1, \ldots, n$ at time $t+1$ sets σ_{t+1}^i to maximize profits given σ_t^{-i}. We extend iterated best responses to the generalized model, which is possible because best responses exist (but may be multivalued) in the generalized model, whereas best responses do not always exist in the pure Bertrand model.

[8] Virtual Bertrand equilibrium satisfies the definition of a sustainable industry configuration given in Baumol et al. (1982) as long as virtual firms are treated as the potential entrants of contestable-market theory.

To begin we abbreviate the notation, writing B_t for B_{σ_t}, C_t for C_{σ_t}, p_t for p_{σ_t}, and Q_t for Q_{σ_t}. We also assume that the profit function $(p-v)D(p)$ is strictly concave in price over $[v, D^{-1}(0)]$.[9] Define $p^*(Q)$ to be a price $p \geq v$ that maximizes

$$\Pi(p) = (p-v)[D(p) - Q],$$

for any $Q \geq 0$. The function Π may be interpreted as the total profits earned by firms that play the Bertrand strategy given that firms that play the Cournot strategy set total quantity Q. By the Maximum Theorem, p^* exists and is continuous over $[0, \infty]$. Note also that $p^*(0)$ is the price that would be charged by a profit-maximizing monopolist.

We make the *switching assumption* that a firm switches from a Cournot to a Bertrand strategy or *vice versa* if and only if the firm earns strictly greater profits by switching. This assumption captures the idea that firms' types do not change without reason; technically, it aids in establishing convergence because it rules out situations in which a firm changes type simply to attain the same level of profits that would be attained if the firm did not change type.

It is known that iterated best responses sometimes do not converge when the market contains three or more firms;[10] indeed, it is difficult to characterize general conditions under which dynamics converge to Cournot equilibrium in the pure Cournot model when $n \geq 3$.[11] Similar difficulties arise in the generalized model here, with the additional complication that best responses are not always single-valued here. Best responses are unique under the switching assumption and a fairly broad convergence result can be established in the duopoly case, however, as long as the initial state satisfies the mild conditions that $Q_0 < D(v)$ and $p_0 > v$. Let p_0^h denote the price initially set by the firm setting the higher price when both firms initially play Bertrand strategies. We prove in the appendix:

Theorem 2. *Assume that $n = 2$ and demand is concave. Then:*

(a) *Dynamics converge to Cournot equilibrium from initial states $\sigma_0 \in \Sigma$ under which $\#B_0 = 0$, or $\#B_0 = 2$ and either $p_0 > p^*(0)$ or $v < p_0 \leq p_0^h \leq p^*(0)$.*

(b) *Dynamics converge to virtual Bertrand equilibrium from initial states $\sigma_0 \in \Sigma$ under which $\#B_0 = 1, p_0 > v$, and $Q_0 < D(v)$, or $\#B_0 = 2$ and $v < p_0 < p^*(0) < p_0^h$.*

Remark: The only initial states not covered by theorem 2 have either a firm playing the Bertrand strategy with price less than or equal to v, or playing the Cournot strategy with quantity greater than or equal to $D(v)$.

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[9] Concavity of profits is equivalent to $2D'(p) + (p-v)D''(p) < 0$. Clearly, sufficient conditions for concavity of profits are that demand slopes down and is either linear or strictly concave ($D''(p) \leq 0$).

[10] See Theocharis (1959).

[11] See Gabay and Moulin (1980, p. 285).

5 Discussion

The result of the preceding section is that iterated best responses converge, in a generalized duopoly setting in which firms choose among Cournot and Bertrand strategies, either to Cournot equilibrium or to virtual Bertrand equilibrium. An implication is that the only dynamic outcome in which both firms in the market have positive sales is Cournot equilibrium. Another implication is that traditional Bertrand equilibrium is *not* an outcome of iterated best responses given generalized strategy sets; neither are equilibria in which one firm plays a Bertrand strategy, one firm plays a Cournot strategy, and both firms have positive sales.

Virtual Bertrand equilibria provide a possible interpretation of "potential competition." Namely, the virtual firms in such equilibria are like potential competitors at the industry doorstep that sell nothing but stand ready to sell at price equal to marginal cost, which keeps the market price equal to marginal cost. A loose analogy might be to the price-setting auctioneer of the Walrasian model. Just as such an auctioneer is like an invisible hand that is needed to support Walrasian equilibrium, at least one virtual Bertrand firm is needed to support virtual Bertrand equilibrium. A difference is that the auctioneer is a theoretical fiction exogenous to the Walrasian system, whereas virtual Bertrand firms may arise endogenously in the model here.

The results here also suggest that Cournot's prediction about equilibrium may sometimes be preferred over Bertrand's predictions. Specifically, virtual Bertrand equilibrium is not robust to removal of the virtual firms from the market. Thus equality between market price and marginal cost relies under iterated best responses on the presence in equilibrium of a virtual firm, or potential competition. This suggests that Cournot's predictions should be preferred over Bertrand's predictions if, in the case of a particular market, there is reason to believe that no virtual firm is part of the market.

Appendix: Proof of Theorem 2

A preliminary lemma is that Cournot and Bertrand strategies are equivalent for firm i in the limited case in which *all* firms other than i play Cournot strategies (the proof is trivial and is omitted):

Price-quantity equivalence lemma. *For any i, suppose all firms except firm i play Cournot strategies. Then for any Bertrand strategy firm i might play, there is a Cournot strategy that yields the same profits, and for any Cournot strategy firm i might play, there is a Bertrand strategy that yields the same profits.*

We now prove theorem 2:

Part (a). If $\#B_0 = 0$, then the price-quantity equivalence lemma implies $\#B_t = 0$ for all t, so the dynamic path from σ_0 coincides with the path that arises when strategy sets are restricted to contain only Cournot strategies. Because D is concave and downward sloping, D^{-1} is also concave and downward sloping, so theorem 4.1 and its applications in section 5 of Gabay and Moulin (1980) imply that dynamics converge to Cournot equilibrium. Assume $\#B_0 = 2$. If $p_0 > p^*(0)$, then each firm plays $p^*(0)$ at $t = 1$. The concavity of D implies that Π is concave, so Π is monotonically increasing over $[v, p^*(0)]$. If $v < p_0 \le p_0^h \le p^*(0)$, then by (1) and (2), each firm earns greater profits by being a price taker at the opponent's price than by undercutting the opponent's price. Thus each firm plays the Cournot strategy with quantity equal to that demanded at the opponent's price. Therefore if $\#B_0 = 2$ and either $p_0 > p^*(0)$ or $v < p_0 \le p_0^h \le p^*(0)$, we have $\#B_t = 0$ for all $t \ge 2$, and the dynamic path from σ_2 coincides with the path that arises when strategy sets are restricted to contain only Cournot strategies. The argument above then implies convergence to Cournot equilibrium.

Part (b). From the switching assumption, if $\#B_0 = 2$ and $v < p_0 < p^*(0) < p_0^h$, then it is optimal for the firm playing p_0 at $t = 0$ to undercut the opponent's price by playing $p^*(0)$ at $t = 1$, and it is optimal for the firm playing p_0^h at $t = 0$ to act as a price taker at the opponent's price p_0 and to set quantity $D(p_0)$ at $t = 1$. Thus $\#B_1 = 1$, $v < p_1 \le p^*(0)$, and $Q_1 < D(v)$. If $\#B_0 = 1, p_0 > p^*(0)$, and $Q_0 < D(v)$, then the firm playing p_0 at $t = 0$ plays $p^*(Q_0)$, which is the price that maximizes $\Pi(p)$ given that the opponent set quantity Q_0, and the firm playing Q_0 at $t = 0$ plays quantity $D(p^*(0))$ at $t = 1$, so again $v < p_1 \le p^*(0)$ and $Q_1 < D(v)$. It therefore suffices to consider initial states with $\#B_0 = 1, v < p_0 \le p^*(0)$, and $Q_0 < D(v)$. Assume $B_0 = \{1\}$ and $C_0 = \{2\}$. It follows from the price-quantity equivalence lemma and the switching assumption that $B_t = \{1\}$ and $C_t = \{2\}$ for all t. Suppose at t that $v < p_t \le p^*(0)$ and $Q_t < D(v)$. Then firm 1 sets $p^*(Q_t)$ and firm 2 sets $D(p_t)$, or

$$p_{t+1} = p^*(Q_t), \tag{3}$$

$$Q_{t+1} = D(p_t). \tag{4}$$

By induction, $v < p_t \le p^*(0)$ and $Q_t < D(v)$ for all t, so (3) and (4) specify the paths of firm 1's prices and firm 2's quantities, and imply $p_{t+1} = p^*(D(p_{t-1}))$. By the definition of $p^*(Q)$, it follows that $D(p_{t+1}) - D(p_{t-1}) \ge 0$ and hence $p_{t+1} \le p_{t-1}$ for $t \ge 2$. Therefore the two subsequences $\{p_{2t}\}$ and$\{p_{2t+1}\}$ are monotonically decreasing. Let p denote the limit of $\{p_{2t}\}$. The continuity of p^* and D implies $p = p^*(D(P))$, so $p = v$. Similarly, $\{p_{2t+1}\}$ converges to v. Thus $p_t \to v$. Because $Q_{t+1} = D(p_t)$, we have $Q_t \to D(v)$. □

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