

A Second-Best Mechanism for Land Assembly

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Abstract

Land can be inefficiently allocated when attempts to assemble separately-owned pieces of land into large parcels are frustrated by holdout landowners. The existing land-assembly institution of eminent domain can be used neither to gauge efficiency nor to determine how to compensate displaced owners adequately. We take a mechanism-design approach to the assembly problem, formalizing it as a multilateral trade environment with perfectly complementary goods. We characterize the least-inefficient direct mechanism that is incentive compatible, self-financing, protects the property-rights of participants, and does not assume that participants have useful information about the subjective valuations of others. The second-best mechanism, which we call the Strong Pareto (SP) mechanism, utilizes a second-price auction among interested buyers, with a reserve sufficient to compensate fully all potential sellers, who are paid according to fixed and exhaustive shares of the winning buyer's offer. It may also internalize local externalities. While the SP mechanism only approves efficient sales, efficiency is not sufficient for sale—even with competitive bidding—because the auction reserve may exceed the aggregate seller valuation. The inefficiency of the second-best mechanism implies a Myerson and Satterthwaite (1981)-style impossibility theorem. We propose a criterion that encompasses concern for both efficiency and the rights of property owners to evaluate the relative performance of assembly mechanisms and the efficiency cost of strict adherence to individual rationality. In a simple example, we compare the expected outcome of the SP mechanism with two alternatives: a plurality mechanism based on SP, but with a lower reserve that is only high enough to fully compensate a plurality of owners and a stylized model of eminent domain.

Keywords: Land assembly, assembly problems, complementary goods, holdout, property rights, mechanism design, desirable properties, impossibility theorem, second-best characterization, SP mechanism, second-price auction, just compensation, local externalities, public-private partnerships

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1 Introduction

Land-assembly projects are frequently delayed or blocked by holdout landowners attempting to capture a greater share of the gains from trade, leading to fragmented and inefficient land use. The problems inherent in land-assembly exemplify a well-known market failure. Economists, at least since Cournot (1838), have understood that attempts to assemble complementary goods or resources can be plagued by holdout. A collection of adjoining parcels of land can be thought of as a single good with fragmented ownership—providing multiple parties the right to exclude—and thus subject to the tragedy of the anticommons (Michelman 1967, Buchanan and Yoon 2000, Heller 1998, Parisi, Schulz, and Depoorter 2005).¹ Apart from land-assembly, holdout problems also plague intellectual property, corporate acquisitions, debt restructuring with multiple creditors, and wage negotiations.² A mechanism that improves upon existing land-assembly institutions might beneficially be applied to any multilateral-trade environment featuring strong complementarities.

The holdout problem is used to justify eminent domain, the legal power of the state to expropriate private property without the owner’s consent.³ However, three connected failings bedevil the use of such public-sector interventions. Firstly, the US and other Constitutions require that owners of compulsorily-acquired property receive ‘just’ compensation. As existing owners are likely to value their property higher than the market, a premium is justified: but how much? Second, the efficiency of a forced transfer of ownership of the assembled

¹ For an interpretation of the hold-out problem as a Prisoners’ Dilemma game, see Miceli and Segerson (2007); see also Menezes and Pitchford (2004). On private ‘takings’ and hold-outs: Nosal (2007), Hellman (2004), and Alpern and Durst (1997).

² See, for example, Heller and Eisenberg (1998), Hazlett (2005), Kieff and Paredes (2007), Geradin, Layne-Farrar, and Padilla Blanco (2008), or Graff and Zilberman (2001) on intellectual property; Burkart and Panunzi (2006) and Van der Elst and Van den Steen (2006) on corporate acquisitions; Miller and Thomas (2006), Hege (2003), Datta and Iskandar-Datta (1995), and Brown (1989) on debt restructuring; and Houba and Bolt (2000), van Ours (1999), Gu and Kuhn (1998), and Cramton and Tracy (1992) on wage negotiations.

³ In the 1980s, the city of Detroit used its power of eminent domain to assemble a large plot of land, and compensated the displaced property-owners at ‘fair market prices’. The city then resold the land cheaply to General Motors, as site for an auto assembly plant. Subsequently, New London, Connecticut, forced a land assemblage, which was leased at very favorable terms for the private development of condominiums and luxury hotels. Heller and Hills (2008) and Lehavi and Licht (2007) give the factual background to the events in Poletown, Detroit, and New London.

land cannot be judged by the usual market tests. And third, the new land-use may generate significant local spillovers, with implications for both equity and efficiency, as well as for the political economy of planning approvals.

What institutions can we design to facilitate the efficient allocation of land, while preserving the rights of owners to a greater degree than under eminent domain? In this paper, we examine how direct assembly mechanisms—used to determine whether a particular assembly will be completed and with what transfers—might mitigate the inherent holdout problem while protecting property rights. We propose the Strong Pareto (SP) mechanism, which 1) implements truthful revelation of private valuations as a dominant strategy for owners and interested buyers; 2) achieves individual rationality by guaranteeing adequate compensation for displaced owners; 3) is self-financing; and 4) does not rely on knowledge of participants' subjective beliefs about the valuations of others.

While the SP mechanism only approves sales that are efficient, it fails to approve some transfers that would be efficient. However, we prove a Myerson and Satterthwaite (1981)-style impossibility theorem for multilateral-trade environments with perfectly-complementary goods and establish the SP mechanism as the second-best mechanism that fully protects property rights. Our main result shows that no mechanism meeting the four above conditions can improve upon the efficiency of the SP by realizing in expectation a greater share of the potential gains-from-trade.

The potential welfare gains from improved land-assembly institutions are large. Given the cultural and legal importance of property rights in Western societies, the justification of eminent domain suggests just how sizeable the inefficiencies due to assembly holdouts are perceived to be. The landmark *Kelo v City of New London* (2005) ruling by the US Supreme Court affirmed the right of governments to use eminent domain to assemble property for private developers. But it initiated a firestorm of concern about the unchecked limits of governments to contravene private property rights. The frequency with which eminent domain is exercised—thousands for times each year (Berliner 2006)—and the public outrage over the *Kelo* ruling suggest that the 'demoralization' costs (Michelman 1967) imposed on

under-compensated owners are significant.⁴

In Section 2 we offer a non-technical description of one acceptable mechanism, the SP mechanism. Owners of properties that are to be assembled as well as property owners within a declared zone surrounding these, would be required to participate in the mechanism. It requires a single, second-price auction of all the relevant properties taken as a whole, with each individual owner nominating the minimum price required for his or her own property. The only element of government compulsion is that the designated property owners must nominate their reservation prices. If a sale is made, then acquisition is approved, as are the broad outlines of the proposed re-development.

Any auction proceeds are distributed to the various former owners, according to fixed and exhaustive shares. The auction is to have a secret reserve, such that the aggregated property will sell only if the second-highest bid is at least sufficient to pay the reservation prices of all the individual owners. The share mechanism ensures that all owners receive at least their nominated reservation prices, if a sale occurs. Because the size of any non-zero payment to a property-owner is independent of the owner's nominated reservation price, the SP mechanism implements truthful revelation as the dominant strategy. Although the land-owners' participation in the mechanism is involuntary, the attractive characteristics of fairness and efficiency may mean that the SP mechanism is more politically-acceptable than is the use of eminent domain itself, for the assembly of property for those public purposes that are to be carried out by profit-seeking firms.⁵ The adherence to individual rationality also leaves open the door to the use of the mechanism in purely private dealings. Furthermore, land re-development almost invariably produces local externalities, with consequences for equity as well as efficiency. The proposed SP mechanism can internalize local externalities

⁴ Epstein (1985) is concerned that some landowners, namely those whose land is condemned must bear a disproportionate cost of a public project. If the project has wide-spread community benefits, the entire community should share that cost proportionately rather than have the lion's share fall on a few unfortunate landowners.

⁵ A main criticism of the use of eminent domain, for what have been called 'economic development takings', is that the displaced property-owners and others, especially the poor and the weak, have been under-compensated, while powerful interest groups have been enriched. Somin (2007) argues that a 'categorical ban on economic development takings is the best solution to the problems Poletown and other similar decisions created.'

if properties surrounding the assembly are included in the single package to be auctioned.

In proposing the SP mechanism, we heed the call of Lehari and Licht (2007) and Heller and Hills (2008) for an auction-based institution to supplant eminent domain. Shavell (2007) proposes a mechanism that in some respects is similar in spirit to the SP mechanism. Landowners state their reserve prices, but unlike SP, potential holdout problems are solved by the public exercise of eminent domain. Two other kinds of mechanism, proposed by Kominers and Weyl (2010) and Plassmann and Tideman (2010), directly address the same general problem. Our approach is to favor property-rights over efficiency, while the other two do the opposite. Like the SP mechanism, the concordance mechanisms of Kominers and Weyl (2010) use exogenously-assigned shares, the assignment of which can be based upon public information—to the extent that it is available—to shore up the mechanism’s ability to pursue its non-constrained objective.⁶ There is no reason why the concordance mechanisms of Kominers and Weyl (2010) cannot satisfy our personal-knowledge condition, though the effectiveness of the specific approach favored by the authors to refunding tax revenue (so as to maintain budget balance and further compensate sellers) depends upon the reliability of the available information. Although the self-assessment mechanism of Plassmann and Tideman (2010) features similar tax-refunds (which need not depend upon information, but can exploit information to avoid under-compensation), the government’s choice of the assessment-tax rate depends heavily upon the knowledge of participants’ beliefs.

In Section 3, we define the assembly problem, characterizing it as a multilateral trade environment with perfect complementarities among the goods offered for sale. Our key observation is that this environment may be viewed as the juxtaposition of a standard single-unit auction problem, namely selecting a buyer from among the interested, and a dichotomous public-goods provision decision: whether or not to approve the sale of the assembled properties at the offered price. An important class of assembly mechanisms, which we call separable, combine an auction for the buyers with a public-goods mechanism for the sellers and the winning buyer.

⁶ For the SP mechanism: efficiency; for concordance mechanisms: protection of property rights.

We then delineate our desiderata for an acceptable assembly mechanism. Most are standard—namely, incentive compatibility, self-financing, and individual rationality. Furthermore, we find deeply questionable, even paradoxical, that any mechanism designed to elicit truthful private valuations should itself be based on the assumption that outside knowledge of such valuations is available through some process other than the operation of well-specified mechanisms. Thus, we also include the less-conventional private knowledge condition, which specifies that a mechanism should in no way rely upon knowledge of the distributions from which the valuations of the participants (both property owners and interested buyers) are drawn, nor should it rely upon knowledge of participants’ subjective beliefs about the valuations of other participants. This is a tighter restriction than is common in the literature, which usually assumes that at least the support of the distribution of private values is common knowledge.⁷ A primary goal of an assembly mechanism is to resolve the holdout problem. Recognizing the difficulty of achieving full efficiency *ex post*, while simultaneously satisfying all of the above conditions, we adopt those conditions as strict requirements of an acceptable mechanism and treat efficiency as an objective to be pursued.

We prove our main result—that no acceptable mechanism can improve upon the expected efficiency of the SP mechanism—in Section 4. We define the expected efficiency of a mechanism as the expected net gains-from-trade captured by the mechanism, normalized by the expected potential gains-from-trade. The second-best theorem relies on two intermediate results. First, for any non-separable acceptable mechanism, there exists a separable acceptable mechanism that is at least as efficient. This allows us to restrict attention to separable mechanisms. Second, the most efficient mechanism features a reserve. Thus, we may restrict attention to acceptable mechanisms that feature auction reserves and judge the relative efficiency of such mechanisms according to whichever has the lower reserve. Standard theory guarantees that the optimal buyer-side mechanism is a second-auction of the assembled properties.

⁷ See, for example, Myerson and Satterthwaite (1981) and Williams (1999).

While insisting on the merits of our requirement of individual rationality and our stringent assumption about private knowledge, we recognize that some degree of violation of ‘just compensation’ for the loss of private property rights is deemed acceptable by society—hence the justification for eminent domain.⁸ Thus, it is important to understand what gains in efficiency can be generated by violating individual rationality. In Section 5, we propose an index of expected-social-value that encompasses concern both for efficiency and for the rights of property owners. It evaluates the expected efficiency of a mechanism, while penalizing it for under-compensating displaced owners.

Then, with a simple example we compare the expected outcome of the SP mechanism with two alternatives: a plurality mechanism based on SP, but with a lower reserve that is only high enough to fully compensate a plurality of owners and a stylized model of eminent domain. Rather than provide a definitive judgment regarding the relative desirability of the mechanisms, the example instead serves to illustrate how institutions for the assembly of complementary goods may be evaluated against each other and how those evaluations depend crucially upon the price society is willing to pay in expected efficiency to avoid the ‘demoralization costs’ imposed on property owners. Section 6 concludes.

2 The Strong Pareto Mechanism

Recent papers by Lehari and Licht (2007) and Heller and Hills (2008) propose an auction of the property assembled through eminent domain for transfer to the private sector. Building on their work, we specify an auction-based mechanism—the ‘Strong Pareto’ (SP) mechanism—which ensures that affected landowners are fairly compensated and that only efficient projects are undertaken. The SP mechanism could be used in public-private partnerships for urban renewal, toll roads, ports and port-side facilities. In these projects, the public sector could use its powers of eminent domain and planning approval, but the private sector becomes responsible for building, owning and operating facilities and structures on

⁸ Michelman (1967) offers a stimulating discussion of the ethical and other arguments for compensation.

the assembled land, for profit. In such situations, the SP mechanism offers an alternative to the use of eminent domain. However, the SP mechanism can be employed only when the auction elicits competitive bidding from private-sector players seeking ownership and control of the assembled parcel of land and the concomitant planning permissions. Although our focus is on land, the mechanism could be used for the assembly of any complementary assets (like patents, stocks and shares, and physical rights-of-way).

To set the background: in pursuit of a public purpose, government has identified a number of properties, with various private owners, as being suitable for assembly into one parcel for redevelopment and use by the private sector.⁹ This section provides an intuitive description of the SP mechanism with an explanation of why it is both individually rational for landowners to participate and to reveal truthfully their personal values.

Envision an expanse of land (100 acres perhaps) subdivided into n (e.g, 100) parcels (0.25 to 2 acres each, say) individually owned by $n \geq 2$ different landowners. Besides owning L_i acres with market value MV_i , each landowner is endowed with x_i of money. Landowner welfare is represented functions $U_i(\cdot)$. The personal value of each parcel, v_i^* , is the amount of money that would just fully compensate a landowner for loss of property.¹⁰ This value is defined implicitly by

$$U_i(L_i, x_i) = U_i(0, x_i + v_i^*).$$

The SP mechanism offers the assembled n plots of land, to be sold as a whole to the highest bidder in second-price auction. Before the auction each landowner is allocated (and informed of) a share, $\alpha_i > 0$, $\sum_{i=1}^n \alpha_i = 1$ of the potential sales price and asked to announce the smallest price, v_i , he would be willing to accept for his parcel. The landowners' value rep-

⁹ For simplicity, we treat all claims over property, including access and use, as though ordinary claims to physical ownership.

¹⁰ It is often asserted that market value must be less than personal value. The reasoning is convincing: if it was not so, the property would have already been sold. Likely, it is not quite so simple, first it implies that people know the market value of their property continuously and second that transfer of ownership is immediate. Certainly, both implications are untrue. Most people are not "up-to-date" on a properties current market value, and even if they are, what is known is the assessed value - the best guess by trained appraisers based on the sale price of comparable properties. Second, taxes and fees drive a wedge between the market sale price and what the owner receives. We ignore both complications.

representations are used to determine the auction reserve—the lowest price at which assemblage ownership will be ceded to a bidder. The reserve, known only to a disinterested broker, is the largest of the ratios of announced landowner values to landowner shares: define a landowner-specific reserve as $r_i = \frac{v_i}{\alpha_i}$, then the auction reserve is $r = \max\{r_i\}$. This auction reserve remains unknown to both landowners and bidders, even at the conclusion of the auction. The only thing the participants know is whether or not the auction is successful (results in a sale); and, if so, the size of the payment made by the winning bidder. If the second highest bid does not exceed the reserve, then landowners retain ownership. Landowner i is *pivotal* if $r_i = r$.

There are $m \geq 2$ potential buyers (indexed by j) who value the assembled properties at w_j^* . For convenience the bidders values are indexed in descending order, with w_1 the highest value.

We assume that every landowner has a notional “subjective” probability density over the value of the winning bid. The individual cumulative density function is $F_i(\cdot)$. The notional distributions are not commonly known, but each is well-behaved and continuous. Furthermore, for each, the expected value exists and is finite.

Section 4 demonstrates that the SP auction is the most efficient amongst mechanism that are individually rational, self-financing, and incentive compatible, and when individual values and notional densities are not known to other participants or to the mechanism designer. Here we offer an intuitive explanation of the results. Because SP is a separable mechanism, the motives of the sellers (landowners) are independent of those of the buyer (bidder) and can be analyzed separately.

The sellers (landowners): If the auction is unsuccessful (because the second price fails to meet or exceed the reserve), the landowners retain their properties and continue to enjoy $U_i(L_i, x_i)$. The process allows each to announce v_i and guarantee at least $U_i(L_i, x_i)$ if the auction is successful. Thus, entering the SP assemblage entails no loss in welfare and the possibility of a welfare improvement. It is thus rational for every landowner to participate. It is straight-forward to demonstrate that announcing v_i^* , the true value, is best irrespective

of the announcements of the other landowners. In other words, truth-telling is a dominant strategy. If v_i^* is i 's announced value and it turns out that i is pivotal, then he will receive at least v_i^* if the auction is successful; and more, if total transfers exceed the reserve. However, if i announces a value that turns out to be pivotal but is smaller than v_i^* , then there is a chance that he may be obliged to cede his property at less than its personal value. And if i 's potentially pivotal announcement, v_i , is larger than v_i^* , this may result in an unsuccessful auction even though i would have enjoyed a welfare gain if he had made a truthful announcement (that is, the second-highest bid was larger than $\frac{v_i^*}{\alpha_i}$ but smaller than $\frac{v_i}{\alpha_i}$).

The Bidders: SP adopts the standard second-price Vickrey auction for the buyers. It is well established that this is the only mechanism that assures that, if there is a successful bid, the property is acquired by the agent with the highest value. Therefore, efficient selection amongst bidders is assured. Because the SP mechanism insists that a transfer of ownership is self-financing, the second highest bid in a successful auction must be at least as large as the auction reserve.

The SP mechanism provides participants no incentive to misrepresent their valuations and it approves only efficient sales. However, efficiency is not sufficient for sale for two main reasons. First, because the winning buyer pays the second price, the buyer's offer may be below the auction reserve even if her value is above it ($b < \sum v_i^* \leq r < w_{max}^*$). However, as the number of interested buyers increases, competition among them drives the buyer-offer/second-price up to the maximum buyer value, eliminating this source of inefficiency.¹¹ Second, unless the SP shares are distributed in a manner that is exactly proportional to personal values, the auction reserve will be too high, exceeding the aggregate seller value. Because the offered land parcels are complementary—in contrast to the buyer side—as the number of sellers increases, the probability that an efficient sale will fail (i.e. when $\sum v_i^* < b < r$) tends towards 1.¹²

¹¹ Both of the dual oligopoly problems considered by Cournot are reflected in the SP mechanism. Buyers of land can be thought of as sellers of a good, namely money, which is perfectly substitutable, thus, efficiency increases with the number of buyers. However, as the number of seller of the complementary land parcels increases, efficiency suffers.

¹² This is a well known result, dating back to Cournot (1838) and established more recently and more rigorously

But a mechanism that ensures efficiency itself requires foreknowledge of individual values and/or notional densities. The requirement is overly restrictive and practically unrealistic.¹³

In spite of its shortcoming, the SP mechanism is desirable in a number of ways. Because the amount paid by a successful buyer equals the payments to displaced landowners, it is self-financing. Because the worst result for both the landowners and the potential buyers is the status quo ante, irrespective of their risk preferences, it is rational for both sellers and buyers to participate and truthful revelation of personal value is a dominant strategy. Because a buyer must pay, an amount that is at least equal to the sum of individual landowner values, all successful ownership transfers are efficient. These are all achieved without the knowledge by anyone - sellers, buyers or mechanism designers - of anyone else's values and/or subjective distributional beliefs. In fact, in the sections that follow we prove the following:

SP is Second Best *There is no assembly mechanism with greater expected efficiency than the SP mechanism that implements truthful revelation as a dominant strategy; is self-financing; does not depend on knowledge of individual preferences and/or subjective beliefs; and that fully respects property rights.*

3 The Assembly Problem and Assembly Mechanisms

Definition 1. An *assembly problem* is defined as

- n sellers (indexed by i) each endowed with a single good, with private and independently drawn valuation v_i^*
- m buyers (indexed by j) with private and independently valuation w_j^* for the assembled package of n goods, and zero value otherwise

An *assembly mechanism* is a direct mechanism that takes announced valuations from all sellers and buyers, $(v, w) = (v_1, \dots, v_n, w_1, \dots, w_m)$, and determines whether the assembled

by Bergstrom (1978) , Mailath and Postlewaite (1990) , Kominers and Weyl (2010).

¹³ If the operator of the mechanism knew individual values or notional densities, then simpler and more efficient mechanisms may be implementable.

items will be sold, to which buyer, and the accompanying transfers. The following is a general definition.

Definition 2. An *assembly mechanism* (Y, X) consists of

- an outcome function, $Y : \mathbf{R}^{m+n} \rightarrow \{0, 1, \dots, m\}$, that determines whether there is a sale and to which buyer, as a function of the vector of announced valuations, (v, w) .
- a transfer function, $X : \mathbf{R}^{m+n} \rightarrow \mathbf{R}^{m+n}$, which specifies the transfers to and from buyers and sellers, as a function of (v, w)

The following notations will be used

Definition 3. The *buyer transfer function*, $X^b : \mathbf{R}^{m+n} \rightarrow \mathbf{R}^m$, denotes the m components of X that specify the buyers' transfers.

Definition 4. The *seller transfer function*, $X^s : \mathbf{R}^{m+n} \rightarrow \mathbf{R}^n$, denotes the n components of X that specify the sellers' transfers.

Definition 5. The *winner function*, $Y^b : \mathbf{R}^{n+m} \rightarrow \{1, \dots, m\}$, specifies the winning buyer, i.e. which buyer gets the sale, conditional on sale.

Definition 6. The *sale function*, $Y^s : \mathbf{R}^{n+m} \rightarrow \{0, 1\}$, specifies whether or not the mechanism approves an assembly, as a function of the announced values v and w .

Note that any assembly mechanism (Y, X) , can be expressed as $Y(v, w) = Y^s(v, w)Y^b(v, w)$ and $X(v, w) = (X^s(v, w), X^b(v, w))$.

3.1 Properties of a Desirable Mechanism

A desirable mechanism has the following properties.

Requirement 1 (Self-financing (SF)).

$$\sum_i X_i^s(v, w) + \sum_j X_j^b(v, w) \leq 0$$

A stricter version of this is to require budget balance.

Requirement 2 (Balanced Budget (BB)).

$$\sum_i X^s(v, w) + \sum_j X^b(v, w) = 0$$

For a given assembly mechanism, define $V_i^o \subseteq \mathbf{R}$ to be the set of optimal messages for seller i given i 's beliefs about the behavior of others and let v_i^o denote an arbitrary element V_i^o . Similarly, let $W_j^o \subseteq \mathbf{R}$ denote the set of optimal messages for buyer j given his beliefs, and let $w_j^o \in W_j^o$. Full respect for property rights means that as long as a seller behaves rationally, she always receives adequate or just compensation for giving up her property. This is the same as requiring that participation in the mechanism is always rational *ex post*, i.e. no rational seller is ever made worse-off by participating. It also requires that successful buyers not be required to pay more than they value the assemblage and that unsuccessful buyers not be required to pay anything at all.

Requirement 3 (Full Respect for Property (FRP) or *Ex Post* Individual Rationality (EIR)).

For all sellers i and buyers j ,

- $Y(v, w) = 0 \implies X(v, w) \geq 0$ and
- $Y(v, w) = j > 0$ implies
 1. $w_j^* + X_j^b(v, w) \geq 0$
 2. $X_k^b(v, w) \geq 0$ for all $k \neq j$ and
 3. $X_i^s(v_i^o, v_{-i}, w) - v_i^* \geq 0$

A weaker condition is *interim* individual rationality, which only requires that a rationally behaving participant (who knows his or her own value realization) is not made worse-off in expectation.¹⁴

¹⁴ Unless otherwise noted, all expectations are taken over the beliefs of the participant about the behavior of all other participants.

Requirement 4 (Interim Individual Rationality (IIR)).

$$E[X_i^s(v_i^o, v_{-i}, w) - Y(v, w)v_i^*] \geq 0$$

and

$$E[I_{Y(v,w)=j}w_j^* + X_j^b(v, w_j^o, w_{-j})] \geq 0,$$

where I is the indicator function.

Next is a standard Incentive Compatibility condition, which requires that it be a dominant strategy for a participant to truthfully reveal his or her private value.

Requirement 5 (Incentive Compatibility (IC)). For all $v_i, v_i^* \in \mathbf{R}$, $v_{-i} \in \mathbf{R}^{n-1}$, and $w \in \mathbf{R}^n$,

$$X(v_i^*, v_{-i}, w) - Y^s(v_i^*, v_{-i}, w)v_i^* \geq X(v_i, v_{-i}, w) - Y^s(v_i, v_{-i}, w)v_i^*$$

and for all $v \in \mathbf{R}^n$, $w_j, w_j^* \in \mathbf{R}$, and $w_{-j} \in \mathbf{R}^{m-1}$

$$I_{Y(v, w_j^*, w_{-j})=j}w_j^* + X_j^b(v, w_j^*, w_{-j}) \geq I_{Y(v, w_j, w_{-j})=j}w_j^* + X_j^b(v, w_j, w_{-j})$$

Another standard requirement is *ex post* efficiency.

Requirement 6 (Efficiency (EF)). A mechanism (Y, X) is *ex post* efficient if

- $Y(v, w) > 0 \iff \sum v_i^* \leq w_{max}^*$, where $w_{max}^* = \max_j \{w_j^*\}$
- $Y(v, w) = j \implies w_j^* = w_{max}^*$

Efficiency requires that, conditional on sale, the assembly be transferred to a buyer with the maximal value, and that sale occurs if and only if the assembly is worth more to that buyer than to the sellers.

The final requirement is uncommon in the standard literature. As is standard, the true valuations (v^*, w^*) are private information, but we further require that the mechanism not rely on participants knowing the true *distribution* of the valuations of others. While we can assume that all parties hold well-defined subjective (notional) probability distributions

about the valuations and choices of others, we cannot expect these subjective probabilities to be accurate, nor can we expect the agency implementing the mechanism to know the true distribution from which (v^*, w^*) is drawn or the beliefs of the participants about it. Thus, no structure is imposed on participants' beliefs about (v^*, w^*) , except that each participant knows his or her own valuation and that individual valuations are independent.

Requirement 7 (Private Knowledge (PK)). The distributions from which (v^*, w^*) are drawn are not public knowledge.

The PK requirement imposes severe limits on the knowledge of sellers, buyers, and mechanism users. However, while the distribution of private values is not publicly known, it nonetheless exists and has a finite expected value. When the implementing agency has no information about the beliefs of the participants, desired properties of the mechanism must hold for *any* beliefs the participants could have. Thus, all local properties must in fact hold globally. Facts 1 and 2 establish two consequences, the first being that the two individual rationality requirements (FRP and IIR) are equivalent.

Fact 1. *Under PK, IIR \iff FRP*

Proof. (\Leftarrow) Obvious.

(\Rightarrow) Under PK, IIR is satisfied only if $E[X_i^s(v_i^o, v_{-i}, b) - v_i^*] < 0$ for *every possible* set of beliefs i may have about v_{-i} and w . A violation of FRP means that for some seller i , there is some possible outcome in which she is forced to sell at less than her valuation ($X_i^s(v_i^o, v_{-i}, b) - v_i^* < 0$). However, if i 's subjective beliefs place sufficient weight on this outcome, $E[X_i^s(v_i^o, v_{-i}, b) - v_i^*] < 0$, so IIR cannot be satisfied. \square

Furthermore, PK and IIR imply a condition much stronger than FRP, namely that conditional on sale, no seller is ever compensated by less than her *announced* value, v_i . This is noted in Fact 2.

Fact 2. *Under PK, IIR $\Rightarrow X_i^s(v_i, v_{-i}, b) - Y(v, w)v_i \geq 0$*

Proof. Follows from Fact 1. \square

4 The Second-best Mechanism

In this section we examine the properties of mechanisms satisfying SF, FRP, IC, and PK and develop a concept of expected efficiency. We characterize the SP mechanism as maximally efficient among mechanism satisfying these four properties. Because it fails to achieve full efficiency *ex post*, it follows that no mechanism can possibly satisfy SF, FRP, IC, PK, and EF simultaneously.

4.1 Acceptable Mechanisms

Several recent papers propose assembly mechanisms, all of which satisfy SF and IC. In contrast with the approach of Kominers and Weyl (2010) and Plassmann and Tideman (2010), who relax FRP and PK, our approach is to treat FRP and PK as constraints and efficiency as an objective. Thus we call a mechanism that satisfies SF, IC, FRP, and PK an *acceptable* mechanism and we characterize the most efficient acceptable mechanism.

Lemma 1, which establishes some implications of requiring acceptable mechanisms to satisfy FRP and SF.

Lemma 1. *SF and FRP imply that*

- $Y(v, w) = 0 \implies X(v, w) = \mathbf{0}$
- If $Y(v, w) = j > 0$, then $X_{j'}^b(v, w) = 0$ for all $j' \neq j$ and
- $X_i^s(v, w) = \alpha_i(v, w)X_j^b(v, w)$ for some $\alpha(v, w)_i \in (0, 1]$, with $\sum \alpha_i(v, w) \leq 1$.

Proof. Obvious □

The three implications are that there are no transfers in the absence of an approved sale; that transfers are only ever between a successful buyer and the sellers; and that total transfers to sellers never exceeds the buyer offer—the conditional transfer from the winning buyer. These restrictions exclude buyer mechanisms such as all-pay auctions and seller mechanisms involving taxes such as the Vickrey-Clarke-Groves (VCG) mechanism, the straightforward

concordance mechanism of Kominers and Weyl (2010), or the self-assessment mechanism of Plassmann and Tideman (2010).

The PK assumption prohibits a mechanism from relying on knowledge of the distribution of (v^*, w^*) . However, because private values are random variables with a well-behaved, albeit unknown, distribution, mechanisms can be evaluated based on their *expected efficiency* (EE) relative to a first-best ideal. This is defined as the expected net gains-from-trade captured by the mechanism. We normalize this measure by the expected potential gains-from-trade, thereby evaluating performance relative to an ideal mechanism.

Definition 7 (Expected Efficiency).

$$EE(M) = \frac{E[Y^s(v, w)(w_{Y^b(v, w)}^* - \sum v_i^*) + \sum_i X_i^s(v, w) + \sum_j X_j^b(v, w)]}{E[I_{w_{max}^* \geq \sum v_i^*}(w_{max}^* - \sum v_i^*)]}$$

This measure rewards the mechanism for approving valuable sales and penalizes it for failing to approve efficient sales and for approving inefficient sales. It also favors budget-balance by penalizing for tax revenues that are not returned directly to participants. Because our analysis focuses on mechanisms that implement truthful revelation as a dominant strategy, we evaluate $EE(M)$ under the assumption that $(v, w) = (v^*, w^*)$.

We begin our characterization of the second-best acceptable mechanism by showing that it must be separable, meaning that it can be broken down into a mechanism that identifies the winning buyer and buyer offer and a separate mechanism that determines whether or not to approve the sale and how to compensate the sellers. Restricting our attention to separable mechanisms, we then characterize the most efficient acceptable buyer mechanism and seller mechanism. Identifying the optimal buyer mechanism is trivial: standard results from the auction literature require acceptable buyer mechanisms to be second-price auctions. Identifying the optimal seller mechanism is more involved. We first show that the most efficient acceptable mechanism must feature a reserve and then we show that the compensation to the sellers must be proportional to the buyer offer.

4.2 Separability

In this section we define the conditions necessary for a mechanism to be separable, then show that, when considering efficiency among acceptable mechanisms, it is sufficient to restrict attention to separable mechanisms. First, the definition:

Definition 8. A mechanism (Y, X) is *separable* if

1. $Y^s(v, w) = Y^s(v, w')$ whenever $\sum_j X_j^b(v, w) = \sum_j X_j^b(v, w')$
2. $Y^b(v, w) = Y^b(v', w)$ for all v, v' and w such that $Y(v, w)Y(v', w) > 0$
3. $X^s(v, w) = X^s(v, w')$ whenever $\sum_j X_j^b(v, w) = \sum_j X_j^b(v, w')$
4. $X^b(v, w) = X^b(v', w)$ for any v, v' and w such that $Y(v, w) = Y(v', w)$

Our first proposition shows that for any acceptable mechanism (whether separable or not), one can construct another acceptable mechanism that *is* separable with expected efficiency no worse than that of the original.¹⁵

Proposition 1. *If an acceptable mechanism $M = (X, Y)$ is not separable, there exists another acceptable mechanism $\hat{M} = (\hat{Y}, \hat{X})$ with $EE(\hat{M}) \geq EE(M)$.*

It follows from Proposition 1 that no non-separable mechanism has strictly greater expected efficiency than the most efficient, separable acceptable mechanism has. Thus, when considering efficiency among acceptable mechanisms, we can restrict attention to separable mechanisms.

A separable mechanism can be broken down into separate mechanisms for buyers and sellers. The *buyer mechanism* identifies the winning buyer based upon the buyer bids, as well as the resulting transfers for the buyers in the case of either a sale or no sale outcome. This requires that the choice of winning buyer, as well as the conditional transfers depend only on the buyer bids, w . The *seller mechanism*, determines whether or not sale is approved, as well as the accompanying transfers to the sellers.

¹⁵ Proofs that do not appear immediately following the stated Lemma or Proposition are presented in the Appendix.

We now restrict attention to separable, acceptable mechanisms. The approval of a sale for a particular v depends only on the *buyer offer*, defined to be the net transfers from buyers conditional on sale (which depends only on w), and the announced seller values. Thus, we may define the buyer offer, $b(w) \equiv \sum_j X_j^b(w)$ the net transfers from buyers conditional on sale and then write $X^s(v, b(w))$ and $Y^s(v, b(w))$. This allows us to adopt a simpler notation, defining separate mechanism for buyers and sellers that specify the outcome and the transfers only conditional on sale, relying on the fact that there can be no transfers in the absence of a sale (established by Lemma 1).¹⁶

Definition 9. A *buyer mechanism*, (Y^b, X^b) consists of

- a winner function, $Y^b : \mathbf{R}^m \rightarrow \{1, \dots, m\}$, which determines the winning buyer, i.e. which buyer gets the sale, conditional on sale
- a conditional buyer-transfer function, $X^b : \mathbf{R}^m \rightarrow \mathbf{R}^m$, which indicates transfers for buyers conditional on sale

Definition 10. A *seller mechanism* (Y^s, X^s) consists of

- a sale function, $Y^s : \mathbf{R}^{n+1} \rightarrow \{0, 1\}$, which determines whether or not a sale will occur, as a function of the announced seller values, v , and buyer offer, b
- a seller transfer function, $X^s : \mathbf{R}^{n+1} \rightarrow \mathbf{R}^n$, which indicates transfers to sellers as a function of the announced seller values, v , and buyer offer, b , conditional on sale.

It follows from Lemma 1 that for a separable mechanism, we may write the conditional transfer of seller i as some share of the buyer's offer: $X_i^s(v, b) = \alpha_i(v, b)b$, where $\alpha_i(v, w) \in (0, 1]$ and $\sum \alpha_i(v, b) \leq q$. However, the assumption of private knowledge means that a particularly strong condition must be imposed on sellers' compensation in order to maintain

¹⁶ Note that these component functions are not quite the same as those defined for general mechanisms. The outcome functions have fewer arguments, with Y^b and X^b depending only on w , and Y^s and X^s depending only on v and the buyer offer, b . Also, the buyer transfer function X^b specify transfers conditional on sale outcome. As is true for all assembly mechanisms, the general form of any acceptable, separable assembly mechanism (Y, X) may be decomposed, as follows: $Y(v, w) = Y^s(v, b(w))Y^b(w)$ and $X(v, w) = [Y^s(v, b(w)) \cdot (X^s(v, b(w)), X^b(w)) + (1 - Y^s(v, b(w))) \cdot \mathbf{0}]$.

incentive compatibility. As shown in Lemma 2, sellers must never be able to manipulate their own compensation.

Lemma 2. *Under PK, $IC \Rightarrow X_i^s(v_i, v_{-i}, b) = X_i^s(v'_i, v_{-i}, b)$ for all $v_i, v'_i \in \mathbf{R}$.*

Proof. For IC to hold under PK, it must be the case that the expected value of announcing v_i^* is no worse than announcing $v_i \neq v_i^*$ for any realization of v_i^* and for any set of subjective beliefs about the behavior of others. Suppose WLOG that for some $v_i^* \neq v_i, v_{-i}$, and b , $X_i^s(v_i^*, v_{-i}, b) < X_i^s(v'_i, v_{-i}, b)$. If i 's subjective beliefs place sufficient weight on v_{-i} and b , then announcing v_i^* will yield a higher expected payoff, violating IC. \square

Thus, we may rewrite $\alpha_i(v, b)$ as $\alpha_i(v_{-i}, b)$. The next lemma states that unless the $\sum \alpha_i = 1$, and thus the mechanism has balanced budget, there is room for efficiency gains. Thus, we may restrict attention to balanced-budget mechanisms when considering efficiency.

Lemma 3. *Let $M = (Y, X)$ be an acceptable, separable mechanism with $X_i^s(v, b) = \alpha_i(v_{-i}, b)b$ whenever $Y(v, b) > 0$, where $\alpha_i(v_{-i}, b) \in (0, 1]$ and $\sum_i \alpha_i(v_{-i}, b) < 1$ for some (v, b) . There exists another acceptable, separable mechanism $\hat{M} = (\hat{Y}, \hat{X})$ with $\hat{X}_i^s(v, b) = \hat{\alpha}_i(v_{-i}, b)b$, where $\sum_i \hat{\alpha}_i \in (0, 1]$ and $\sum_i \hat{\alpha}_i(v_{-i}, b) = 1$ for all (v, b) (i.e. budget-balanced) that has greater expected efficiency than M .*

Proof. Define \hat{M} as follows: $\hat{Y} = Y$ and $\hat{X}^b = X^b$; if $Y^s(v, b) = 0$ then $\hat{X}^s(v, b) = X^s(v, b) = \mathbf{0}$; if $Y^s(v, b) = 1$ then for each i , $\hat{X}_i^s(v_{-i}, b) = \hat{\alpha}_i(v_{-i}, b)b$, where $\hat{\alpha}_i(v_{-i}, b) = \alpha_i(v_{-i}, b) + \frac{1}{n}[1 - \sum_i \alpha_i(v_{-i}, b)]$. The new mechanism, \hat{M} approves the exact same sales as M , but normalizes the shares that determine sellers transfers to sum to one. The shares, and thus, seller compensation remain independent of v_i , so incentive compatibility is maintained. Furthermore, because the seller shares are exhaustive, the mechanism is budget-balanced. Finally, because for at least one (v, b) for which $Y(v, b) > 0$, there is at least possible sale outcome for which the seller transfers determined by \hat{M} exceed those of M , which holding b constant. This means that \hat{M} must have strictly greater expected efficiency than M . \square

4.3 The Reserve

Next, we show that the most efficient mechanisms will feature a strict cutoff, or reserve, for the buyer's offer, below which sales are rejected and above which sales are approved. We begin by defining what it means for a separable mechanism to feature a reserve.

Definition 11. A separable mechanism $M = (Y, X)$ features a reserve if, for each v , there exists an $r(v)$ such that $Y^s(v, b) = 1 \iff b \geq r(v)$.

Lemma 4 shows that we may restrict attention to mechanisms that feature a reserve, because any separable, acceptable mechanism without a reserve is accompanied by another separable, acceptable mechanism that does feature a reserve and is at least as efficient.

Lemma 4. *If M is a separable, acceptable mechanism that does not feature a reserve, there exists another separable, acceptable mechanism, M' , that does feature a reserve price with $EE(M') \geq EE(M)$.*

Note that efficiency is a necessary condition for sale in any acceptable mechanism, because the buyer's offer alone must be able to fully compensate all sellers at least their truthfully-reported values. Thus, to maintain FRP, in any acceptable mechanism that features a reserve, $r(v) \geq \sum_i v_i$ for any $v \in \mathbf{R}^n$. This leads to Fact 3, which states that because the message space for each individual seller is unbounded above, the set of reserves has no upper bound either.

Fact 3. *For any $v \in \mathbf{R}^n$, there exists a $v' \in \mathbf{R}^n$ such that $r(v) < r(v')$.*

Proof. Choose any v' such that for some i , $r(v) < v'_i$. Then $r(v) < v'_i < \sum_i v'_i < r(v')$. \square

It follows from this fact, that no separable, acceptable mechanism can feature a constant reserve.

Because the second-best acceptable mechanism must be separable, we may proceed by characterizing the optimal buyer and seller mechanism separately. The most-efficient acceptable buyer mechanism will be the one that in general yields the greatest buyer offer for

a given set of announced buyer values, while the most efficient acceptable seller mechanism will be the one that in general yields the lowest reserve from a given set of announced seller values. Standard auction theory results guarantee that the optimal buyer-side mechanism is a second-auction of the assembled properties. BB, FRP and IC require a k th-price auction, and the most efficient of these is the second-price auction ($k = 2$). Thus, the remaining task is to characterize the optimal seller mechanism.

Our main result is established by the next two propositions. Proposition 2 shows the auction reserve must exceed the the minimum-acceptable buyer offer—derived by inflating the seller’s value using her assigned share—for all sellers and that each seller’s share must be completely fixed—manipulable neither by the seller herself, by the other sellers, nor by any buyers.

Proposition 2. *Let $M = (Y, X)$ be an acceptable, separable mechanism with balanced budget and consider a vector of shares $\alpha(v, b) = (\alpha_1(v_{-1}, b), \dots, \alpha_n(v_{-n}, b))$, where $X_i^s(v, b) = \alpha_i(v_{-i}, b)b$ for each i and where $0 < \alpha_i \leq 1$ and $\sum \alpha_i(v_{-i}, b) = 1$. Then $r(v) \geq \max_i \left\{ \frac{v_i}{\alpha_i(v_{-i}, b)} \right\}$ and for each i there exists some $\alpha_i \in (0, 1]$ such that $\alpha_i(v_{-i}, b) = \alpha_i$ for all $v_{-i} \in \mathbf{R}^{n-1}$ and $b \in \mathbf{R}$.*

It follows that we may write $X_i^s(v, b) = \alpha_i b$, where $\alpha_i \in [0, 1]$ and $\sum_i \alpha_i = 1$ and $r(v) \geq \max_i \left\{ \frac{v_i}{\alpha_i} \right\}$. The final Proposition establishes that unless the above inequality is binding, there is room for efficiency gains.

Proposition 3. *Let $M = (Y, X)$ be an acceptable, separable mechanism with balanced-budget and $X_i^s(v, b) = \alpha_i b$, where $\alpha_i \in (0, 1]$ and $r(v) \geq \max_i \left\{ \frac{v_i}{\alpha_i} \right\}$ for all v , but $r(v) > \max_i \left\{ \frac{v_i}{\alpha_i} \right\}$ for some v . There exists a another acceptable, separable, balanced-budget mechanism, $\hat{M} = (\hat{Y}, \hat{X})$ with $\hat{X}_i^s(v, b) = \hat{\alpha}_i b$, where $\hat{\alpha}_i = \alpha_i$ and $\hat{r}(v) = \max_i \left\{ \frac{v_i}{\hat{\alpha}_i} \right\}$ for all v , that has greater expected efficiency than M .*

Proof. Define \hat{M} as follows: $\hat{Y}^b(w) = Y^b(w)$ and $\hat{X}^b(w) = X^b(w)$; $\hat{Y}^s(v, b) = I_{b \geq \hat{r}(v)}$, where $\hat{r}(v) = \max_i \left\{ \frac{v_i}{\hat{\alpha}_i} \right\}$; and where $\hat{X}^s(v, b)$ is defined by $\hat{X}_i^s(v, b) = \hat{\alpha}_i b$, where $\hat{\alpha}_i = \alpha_i$. By

construction, $\hat{r}(v) = \max_i \{ \frac{v_i}{\alpha_i} \}$. Clearly, \hat{M} inherits separability and acceptability from M . It must be the case that either $\hat{r}(v) \leq r(v)$, with strict inequality for some v . Having a lower reserve would increase the expected efficiency by increasing the probability of efficient sale. \square

The resulting class of mechanisms features fixed and exhaustive shares that determine the conditional seller transfers, and are used to inflate sellers valuations to determine individual reserves, the maximum of which is the auction reserve: this combination of features defines the SP seller mechanism. While the agency implementing the mechanism may have prior beliefs about the distributions of valuations, without specifying those beliefs one cannot compare the expected efficiency across different members of the SP family of mechanism corresponding to different share assignments. Thus, we complete the analysis of expected efficiency among acceptable mechanisms with the conclusion that the SP family of mechanisms is at least as efficient as any acceptable mechanism, having established our primary claim:

SP is Second Best *There is no assembly mechanism with greater expected efficiency than the SP mechanism that implements truthful revelation as a dominant strategy; is self-financing; does not depend on knowledge of individual preferences and/or subjective beliefs; and that fully respects property rights.*

The SP mechanism is the least-inefficient mechanism that fully respects property rights and can be used in low information environments. However, while efficiency is necessary for sale, because the second price may be too low, or the reserve may be too high, efficiency is not sufficient for sale. An immediate corollary of this failure is that an ideal (first-best) mechanism is impossible.

Corollary 1. *No acceptable mechanism can satisfy EF.*

While the impossibility of trade with a large number of sellers is well established,¹⁷ as

¹⁷ See for example, Cournot (1838), Sonnenschein (1968), Bergstrom (1978), Mailath and Postlewaite (1990), and

far as we know, this paper is the first to establish the impossibility of an *ex post* efficient, acceptable mechanism for *any* sized assembly problem.¹⁸

5 Evaluation

The impossibility of an ideal mechanism guarantees that policymakers who would like to apply an incentive-compatible direct mechanism to the assembly problem must confront the tradeoff between efficiency, budget-balance, and the rights of property owners. The desirability of a particular mechanism depends upon how closely its performance matches the balance preferred by the relevant decision-maker. While our approach in this paper is to treat individual rationality as a binding constraint, this is likely too extreme. The justification for eminent domain reveals that society is willing to accept some under-compensation in pursuit of social efficiency. Thus, individual rationality (FRP), which requires the unanimous agreement of all sellers (and buyers), is a an obvious candidate for violation.

Other proposed mechanisms, such as the self-assessment mechanism of Plassmann and Tideman (2010) and mechanisms based on the concordance principle of Kominers and Weyl (2010) promote efficiency at the expense of budget-balance and property rights. More generally, many public choice and public policy mechanisms violate individual rationality. If FRP is not required, it is inevitable that some landowners will suffer losses. How how great are these losses and to what degree do they permit increased efficiency? Little is known about the nature of these tradeoffs, which surely depend upon the distribution of valuations, beliefs, and the number of participants. Furthermore, our judgment about which institutions are preferable must depend on how those losses are evaluated.

In this section, we use a simple example to explore the consequences of relaxing the FRP condition and we compare the performance of the SP mechanism to that of a stylized model

Kominers and Weyl (2010).

¹⁸ While the proof of the second-best theorem relies upon the private knowledge condition—the relaxation of which would surely allow greater efficiency—we strongly suspect that allowing informational assumptions more in-line with the rest of the literature, e.g. in Myerson and Satterthwaite (1981) would still lead to impossibility.

of eminent domain. In a constructed assembly problem, we compute the extent to which each institution, on average, realizes the potential efficiency gains and avoids under-compensating sellers. The scenario is sparse and the calculations transparent, to illustrate how to compare SP with the two alternative institutions.

To evaluate performance, we apply a social-value metric that encompasses both concerns for efficiency and for the rights of property owners. The definition of the expected social value of a mechanism simply modifies the definition of expected efficiency (Definition 7) by subtracting the weighted dollar value of under-compensation. As in the efficiency definition, the numerator is normalized by the performance of an ideal mechanism—one which captures all gains from trade and fully protects property rights.

Definition 12 (Expected Social Value).

$$EE(M) = \frac{E[Y^s(v, w)(w_{Y^b(v, w)}^* - \sum v_i^*) + \sum_i X_i^s(v, w) + \sum_j X_j^b(v, w) - \beta \sum_{i \in U} (v_i^* - X_i^s(v, w))]}{E[I_{w_{max}^* \geq \sum v_i^*} (w_{max}^* - \sum v_i^*)]},$$

where U is the set of all owners for whom $v_i^* > X_i^s(v, w)$ and $\beta \geq 0$ is a parameter that reflects the degree to which the policy-maker finds under-compensation undesirable. We calculate expected social value under the assumption of truthful revelation $(v, w) = (v^*, w^*)$.

We emphasize the fact that with this example, we do not claim to provide a definitive judgment regarding the relative attractiveness of the institutions. Rather, we use the example to illustrate the application of the Expected Social Value (ESV) metric to evaluate the costs of strict adherence to FRP and to highlight the strengths and weaknesses of the SP mechanism relative to some plausible alternatives.

5.1 Example: Three Institutions Applied to an Assembly Problem

Consider the sale of three individually-owned parcels as one assemblage, where parcels 1, 2, and 3 are valued at \$1, \$2, and \$3 by their respective owners. Thus, any proposed use of the assemblage with value greater than \$6 represents an efficiency-enhancing transfer of ownership. The individual valuations are private, but the three parcels have the same market

value and there is no public information beyond this that allows the government to estimate the distribution of values.

The SP mechanism prescribes the assignment of shares, α_i , drawn from the unit simplex. We calculate individual reserves, $r_i = \frac{v_i^*}{\alpha_i}$, and the overall auction reserve, $r = \max_i\{r_i\}$, under the assumption that sellers truthfully reveal their values. Sale occurs if and only if $r \leq b$, where b is the buyer's offer, and conditional on sale, each seller is compensated by $\alpha_i b$. As vast simplification, here we assume that the government has drawn the shares $\frac{1}{6}$, $\frac{2}{6}$, and $\frac{3}{6}$, which happen to correspond to the respective sellers shares of the aggregate private value. If these are ideally assigned, the SP auction reserve would be \$6 and the mechanism would produce a fully-efficient result. However a guarantee of this assignment would violate the PK condition, one that we hold most inviolate. Thus, we examine the consequence of a random assignment of these shares.

Contrasted with SP are two alternatives, both of which violate the individual rationality restriction. The first replaces unanimity with a *plurality* rule: here, the reserve is set so that at least two out of the three landowners are fully compensated. The *Plurality* (PL) mechanism uses the same shares as the SP mechanism, but the reserve, r , is defined to be the median of the individual reserves, r_i .¹⁹ As with the SP mechanism, compensation is equal to the product of the share and the final selling price.

The second comparison is with a version of governmental takings, which we call *stylized eminent domain* (SED), with market values as the only guide to the cost-benefit calculation and for compensation. Sale is approved and property is condemned if the alternative use is at least as valuable as the government's determination of the aggregate value of the individual parcels, which in this case is \$6.²⁰ Because we assume that the government correctly

¹⁹ Kominers and Weyl (2010) refer to this as the *.5-plurality mechanism*. They also consider the more general category of *X-plurality mechanisms*, which require a fraction X of owners to be fully compensated.

²⁰ One interpretation of this feature of the SED model is that all three parcels have market value of \$2 and that the government simply uses market value to judge efficiency and determine compensation, as in typical in the application of eminent domain. Though it may seem unusual for the owner of parcel 1 to have a private valuation below that of the market, there are good reasons which this may be the case. For example, individuals do not always know the market value of their home, which fluctuates and requires a costly appraisal to assess. Alternatively, one can interpret the government as taking the market valuations and uniformly applying a standard percentage increase to estimate

determines the aggregate private value of the parcels, the SED process to be perfectly efficient. This glosses over the fact that the actual application of eminent domain surely results in some inefficient transfers and the failure of some efficient transfers and surely biases our evaluation of the institutions towards the SED model. When there is a sale, the proceeds are divided equally among the three owners, based upon the equal market values.

Table 1 displays the reserves that would result for each institution from each possible assignment of the three shares ($\frac{1}{6}$, $\frac{2}{6}$, and $\frac{3}{6}$). Note that, by assumption, SED always features the efficient reserve of \$6, while the SP reserve is inefficiently high unless the shares are assigned perfectly in proportion to private values, which only occurs one-sixth of the time, as shown in row 1. The plurality mechanism has a greater tolerance for poorly assigned shares, but may feature a reserve that is too low (below \$6), as in row 4, as well as a reserve that is too high, as in row 5.

Table 1: Reserves for each institution, as a function of share assignment.

Share			Reserve		
\$1	\$2	\$3	SP	PL	SED
$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{2}$	6	6	6
$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{3}$	9	6	6
$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{2}$	12	6	6
$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{6}$	18	4	6
$\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{3}$	12	9	6
$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{6}$	18	6	6

5.2 Expected Efficiency

In calculating the ESV of each mechanism, we focus on seller behavior, abstracting from buyer-side behavior by drawing the buyer offer, b , from a uniform distribution over the interval $[0, 20]$ as a proxy for the outcome of the buyer-side mechanism. To the extent that

the private valuations.

the buyer offer can be assumed to be the outcome of a competitive bidding process featuring many buyers, b reflects the maximal buyer value and can suitably used to test efficiency.²¹ Because the expected buyer offer (\$10) exceeds the aggregate seller value, sale is efficient on average. Next we calculate the expected efficiency for each institution and for the full-information benchmark, under the assumption that the shares are assigned to each owner with equal probability.

Full-information The benchmark against which we evaluate the other institutions in the full-information case in which the owners' individual valuations are public and efficiency and compensation can be gauged perfectly. In this case, the reserve is always \$6. The probability of a successful bid $1 - 6/20 = 0.7$ and the expected value of the bid, conditional on sale is $\frac{1}{2}(20 + 6) = 13$. Thus, the expected gains-from-trade for the first-best mechanism are: $0 \cdot 0.3 + (13 - 6) \cdot 0.7 = 4.90$.

SP The SP mechanism yields the ideal reserve of \$6 with probability 1/6 and the probabilities of the higher reserves of \$9, \$12, and \$18 are 1/6, 1/3 and 1/3, respectively. The probability of sale is 0.7, 0.55, 0.4, and 0.1, respectively. Conditional on sale, the expected buyer offers for the four possible reserve values are $E[b|r = 6] = 13$, $E[b|r = 9] = 14.5$, $E[b|r = 12] = 16$, and $E[b|r = 18] = 19$. Thus, the expected gains-from-trade for the SP mechanism are:

$$\frac{1}{6}[(13 - 6) \cdot 0.7] + \frac{1}{6}[(14.5 - 6) \cdot 0.55] + \frac{1}{3}[(16 - 6) \cdot 0.4] + \frac{1}{3}[(19 - 6) \cdot 0.1]$$

or

$$\frac{1}{6}4.90 + \frac{1}{6}4.675 + \frac{1}{3}4 + \frac{1}{3}1.3 \equiv 3.36$$

²¹ Otherwise, b may understate the maximal buyer value, and using it as a proxy will overstate the realized gains-from-trade, though by the same amount for each institution. By using b in both the numerator of the efficiency measure and in the normalization term in the denominator, we are essentially holding each institution to the standard of an ideal *seller* mechanism, which is appropriate given our focus on seller behavior.

Plurality If we require only an approval plurality, the reserve achieves the \$6 ideal two-thirds of the time and is \$4 and \$9 one-sixth of the time each. The expected efficiency of the Plurality mechanism is thus:

$$\frac{2}{3}4.9 + \frac{1}{6}4.675 + \frac{1}{6}[(12 - 6) \cdot 0.8] \equiv 4.85$$

Stylized Eminent Domain Since, as the example is constructed, the government determines sale according to the correct aggregate seller valuation of \$6, the expected gains-from-trade for the SED model are the ideal 4.90.

Normalizing the gains-from-trade for each institution by the expected potential gains of 4.90, we arrive at expected efficiencies of .69, .99, and 1.0 for SP, PL, and SED, respectively. Thus, measured by efficiency alone, SP does not compare favorably to the other two institutions. However, these calculations ignore the social cost of individual under-compensation.

5.3 Demoralization Costs

Michelman (1967) used the label “demoralization costs” to indicate the potential damage done to individuals forced to cede their property unwillingly. Epstein (1985) pointed to the “disproportionate cost” that should be considered when property is condemned for public use. These costs, while interesting theoretical constructs, suggest a metric to adjust the efficiency calculation. Indeed, judgment about the relative desirability of institutions may depend upon how these personal costs are evaluated.

Our simple metric identifies these costs with the dollar amount by which an owner is under-compensated for her property and factors them linearly into the social-value calculation. We propose a parameter, β , to represent the ‘price’ that society would be willing to forego in expected efficiency in order to avoid a dollar of under-compensation. The degree of social-concern for the rights of property owners is captured by β : the larger its value, the more the concern. Those who argue that the difference between the dollar value of the public use and the dollar value of the private land holdings is the only calculation relevant

to the cost-benefit analysis would choose $\beta = 0$. Those who see property rights as inviolate would choose $\beta = \infty$. Clearly, the uproar over the Kelo decision indicates that there is some degree of social aversion to forced property losses, or that $\beta > 0$.

The SP mechanism generates no demoralization costs: every landowner receives at least her personal value. While the Plurality mechanism may approve the transfer of the assemblage to a use of *lower* value, which necessarily leads to under-compensation, it may also under-compensate some owners even when the transfer enhances social efficiency. The expected monetary value of under-compensation is \$0.28.²² Finally, eminent domain, irrespective of buyer value, always under-compensates the high-value landowner by \$1 if there is a buyer willing to pay at least \$6 for the assembled properties. Since the probability of sale is 0.70, the expected under-compensation is \$0.70. Normalizing these three amounts by the ideal expected gains-from-trade (\$4.90) yields expected under-compensation measures of 0, 0.06, and 0.14.

Table 2 summarizes the efficiency and under-compensation results for each mechanism or institution. The ESV of the SP mechanism is 0.69, while the ESV of the Plurality mechanism is $0.99 - 0.06\beta$ and the ESV of the SED model is $1 - 0.14\beta$. The order in which society ranks the three institutions clearly depends upon β , the importance placed on property rights. Because the SP mechanism sacrifices efficiency in favor of the rights of owners, for low levels of β it is less desirable than the alternatives examined. However, as shown in Table 2, if a

²² This is calculated as follows. There is a one-sixth chance that the shares will be ideally assigned and there will be no under-compensation, as is shown in row 1 of Table 1. Whenever $6 \leq b \leq 9$, the share assignment displayed in row 2 of Table 1 leads an approved sale for which the owner of parcel 3 is under-compensated by $3 - b/3$. This event occurs with probability $\frac{3}{20}$ and conditional on it occurring, the average under-compensation is \$0.50. Thus, one-sixth of the time there is expected under-compensation of $\frac{3}{40}$. In row 3 case, there is a $\frac{6}{20}$ chance that a sale will be approved with a bid between 6 and 12, in which case the owner of parcel 2 is under-compensated, with a conditional average amount of \$0.50. Thus, there is a one-sixth chance of expected under-compensation of $\frac{6}{40}$. For the row 4 case, the probability of under-compensation is $\frac{14}{20}$, with average conditional under-compensation of $\frac{7}{6}$. For the row 5 case, the probability of under-compensation is $\frac{3}{20}$ and the average conditional under-compensation is 0.25. Finally, for the row 6 case, the probability of under-compensation is $\frac{12}{20}$ and the average conditional under-compensation is \$1. Thus, the expected under-compensation is

$$\frac{1}{6} \left[0 + \frac{3}{40} + \frac{6}{40} + \frac{49}{60} + \frac{3}{80} + \frac{12}{20} \right] = \frac{1}{6} 1.68 = 0.28$$

dollar of under-compensation is penalized slightly more than twice as heavily as a gained dollar of social efficiency then the SP mechanism outshines the (highly stylized and favorably constructed) eminent domain model and, if the penalty for under-compensation is as high as five, then SP compares favorably to the Plurality mechanism as well.

Again we emphasize that this example is not intended to provide a definitive judgment regarding the relative desirability of the mechanisms. Rather, it illustrates how institutions for the assembly of complementary goods may be evaluated against each other and how those evaluations depend crucially upon the price society is willing to pay in expected efficiency to avoid the ‘demoralization costs’ imposed on property owners when their property is forcibly transferred out of their possession.

Table 2: Expected social value for each mechanism and, for the two alternative institutions, the minimum price of under-compensation (β) sufficient to render SP preferable.

	Eff.	Under-comp.	Equalizing β
SP	0.69	0	NA
PL	0.99	0.06	5
SED	1.00	0.14	2.17

6 Conclusion

We have shown that, for the assembly of perfectly-complementary assets, the SP mechanism is the least inefficient mechanism that is self-financing, incentive-compatible, individually rational, and does not rely on information about the support of the distribution of subjective values. Along the way we proved the impossibility of any ex post efficient and acceptable mechanism for assembly problems of any size. In this context, our stylized example suggests that, when it is acceptable to violate property rights in order to improve efficiency, the SP mechanism may be superior to eminent domain and to plurality, even if a relatively-low ‘penalty’ is placed on property rights violations. Although this paper concentrated on land

assembly, the SP mechanism can be applied to many assemblies of complementary assets, real or financial, including possibly to purely-private assemblies (e.g., as an alternative to the law permitting compulsory sale of minority share-holdings; or informal private arrangements for reciprocal violation of patents).

Any incentive-compatible mechanism not only provides an economic answer to a legal puzzle-what is 'just compensation' for property taken and transferred to a private owner-but it also provides an appropriate test of the efficiency of land re-development, subject to one qualification. The consequences of the kind of infrastructure project and urban re-development that require assembly generally extend beyond the boundaries of the development area itself. Some surrounding properties may gain in amenity or market value, e.g., because of the prospect of employment in a new factory, or the convenience of a new shopping center. Others may lose, say, due to noise pollution or congestion. These changes in land values should be included in the test of efficiency. Existing planning and political processes commonly take some account of the interests of the owners of properties or rights in a local zone declared around the development proper, but they are subject to the same failings as eminent domain in judging overall efficiency. A theoretically-appropriate efficiency test includes these externalities.

If local properties subject to possible spillovers are included in the assembled parcel, their external costs and benefits will be internalized through common ownership.²³ Any bidder, when assessing the advantages of (future) marginal expenditure in the assembled area, will consider not only the effects on the value of the assembly itself, but also the effects on the value of land surrounding the assembly. Therefore the application of the SP mechanism should include in the auction properties affected by local spillovers, as this provides the appropriate efficiency test for the assembly. The limitation is that, for practical reasons, an

²³ Similar are 'company towns' in which the owner of, say, a huge mining tenement, establishes, on land that the company owns, a town for workers and those who service their needs. For land grants to railway entrepreneurs, see Pincus (1983). In the absence of land grants, governments have used betterment taxes and other Henry George-like schemes (Starrett 1988). The urban infrastructure of Canberra, Australia, prior to self-government, was largely financed through the development authority's capture of the increased land values that its developmental expenditures induced.

arbitrary line must be drawn between those properties in the widened assembly and those outside.²⁴

For two main reasons, our formally-modelled assembly problem overstates the magnitude of the social inefficiency due to holdout. First, the degree of complementarity within real assemblies is not always perfect because the boundaries may not be fixed and pre-determined. For example, the exclusion from the assembly of a small property at the perimeter of the development area may reduce the value of the assembly slightly, rather than totally. Smaller fragments of an assembly may be easier or cheaper to work around. Moreover, the very fragmentation of the good that exacerbates holdout under perfect complementarity may also reduce the strength of the complementarity itself, thereby mitigating the holdout problem: if a property owner refuses to sell, then the development area may be able to be reshaped (at some cost) to include a close-substitute property not in the area originally targeted. Second, while there is no efficiency-enhancing competition among sellers of perfectly-complementary goods, some substitutability may exist between assemblies, and therefore some room for efficiency-enhancing competition. To the extent that such substitutability does exist, encouraging competition between development areas (and not just developers) may alleviate some of the frustrating limitations of market design highlighted herein and in Kominers and Weyl (2010). For example, if there are a number of feasible routes for a pipeline or a toll road, each requiring the assembly of perfectly-complementary holdings, then competition between the routes may reduce the value of holding out.²⁵

²⁴ Similar ‘zoning’ has been used by governments to limit the number of households that developers must notify; and to which compensation is made for the additional noise created by the extension of airport runways or relaxation of airport curfews (e.g. via subsidized sound-proofing), and the like.

²⁵ Pincus and Shapiro (2008) discuss the application of the SP mechanism to the sale of collectively-controlled water rights in irrigation districts, for a government program to purchase water for environmental purposes.

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A Proofs

Proof of Proposition 1.

Proof. Let $M = (Y, X)$ be an acceptable mechanism that is not separable and note that Lemma 1 applies. The proof proceeds by defining a new mechanism in four iterations, each addressing one of the separability conditions. For condition 1, for each v , we define the outcome of the new mechanism by taking all of the w that yield the same aggregate buyer transfers under the old mechanism, and assigning them all to the highest sale outcome observed in that set. We do this as follows: for a given v , define $W^v(b) = \{w \in \mathbf{R}^m \mid \sum_j X^b(v, w) = b\}$, the set of all w that lead to buyer offer b , and $b^v(w) = \sum_j X^b(v, w)$, the buyer offer that results from w . Thus, $W^v(b^v(w))$ is the set of all w that lead to the same buyer offer as w . Finally, define $\bar{j}(w) = \max_{w' \in W^v(b^v(w))} Y(v, w')$, the highest sale outcome among all of the possible vectors of buyer announcements that lead to the same buyer offer as w . This takes on the value zero if there is never any sale at that level of buyer offer, otherwise it is simply the maximum index of buyers who are successful at the same level of buyer offer. Now, choose any $\bar{w}(v, w) \in \arg \max_{w' \in W^v(b^v(w))} Y(v, w')$ and then define a new mechanism $M_1 = (Y_1, X_1)$ as follows:

$$(Y_1(v, w), X_1(v, w)) = \begin{cases} (Y(v, w), X(v, w)) & \text{if } Y(v, w) > 0 \\ (\bar{j}(w), X(v, \bar{w}(v, w))) & \text{if } Y(v, w) = 0 \end{cases}.$$

M_1 inherits acceptability from M and it satisfies condition 1 by construction. Furthermore, its acceptability means that all approved sales are efficient. Thus, by approving sale for a set of values that contains the set approved by M , M_1 is guaranteed to have expected efficiency no worse than that of M .

For condition 2, we will define the outcome of the next iteration of the mechanism by assigning the sale (conditional on sale) to the buyer who has a highest bid, from the set of all buyers who are ever successful given the existing bids. For each $w \in \mathbf{R}^m$, let $J(w) = \{j > 0 \mid Y_1(v, w) = j \text{ for some } v \in \mathbf{R}^n\}$, the set of all buyers who, given w , are successful for

some vector of announced seller values. Define a new mechanism, $M_2 = (Y_2, X_2)$ as follows:

$$(Y_2(v, w), X_2(v, w)) = \begin{cases} (0, \mathbf{0}) & \text{if } Y_1(v, w) = 0 \\ (\bar{j}, X(v, w_{\bar{j}})) & \text{if } Y_1(v, w) > 0 \end{cases},$$

where $\bar{j} \in J(w)$ and $w_{\bar{j}} \geq w_j$ for all $j \in J(w)$. By always assigning an approved sale to the same buyer, who, among the set of buyers who ever are successful given w , has maximal value, M_2 is guaranteed to satisfy condition 2 (as well as condition 1) and to achieve expected efficiency no worse than that of M_1 . It inherits acceptability from M_1 .

For condition 3, we define the next mechanism by choosing one set of sellers transfers that is implemented for the same v and *some* w with the same $b(w)$, then dividing an remaining funds from the buyer offer equally among the sellers, thereby establishing budget-balance. For each $v \in \mathbf{R}^n$ and $b \in \mathbf{R}$ with non-empty $W^v(b)$, fix some $w_b^v \in W^v(b)$. Define a new mechanism, $M_3 = (Y_3, X_3)$, by

$$(Y_3(v, w), X_3(v, w)) = \begin{cases} (Y_2(v, w), X_2(v, w)) & \text{if } Y_2(v, w') = 0 \text{ for all } w' \in W^v(b^v(w)) \\ (Y_2(v, w), (X_2^s(v, w_{b^v(w)}^v) + d, X_2^b(v, w))) & \text{otherwise} \end{cases},$$

where $d = \frac{1}{n}(b^v(w) - \sum_i X_{2i}^s(v, w_{b^v(w)}^v))$. Given v , the seller transfers implemented by M_3 depend only on the buyer offer, b , so it satisfies condition 3. M_3 inherits acceptability from M_2 as well as conditions 1 and 2. Because it features the same outcome function as M_2 , but is budget-balanced, it has expected efficiency no lower than that of M_2 .

Finally, for condition 4, we define the next iteration of the mechanism by resetting the buyer offer for every v /successful buyer combination to the highest offer ever implemented for that buyer given v . For each $w \in \mathbf{R}^m$ and $j \in \{0, \dots, m\}$, let $V_j^w = \{v \in \mathbf{R}^n | Y(v, w) = j\}$ and $b_j^w = \inf_{v \in V_j^w} X^b(v, w)$. Thus, b_j^w is the highest buyer offer associated with successful buyer j when the announced buyer values are w . Now define a new mechanism, $M_4 = (Y_4, X_4)$, as follows: $Y_4(v, w) = Y_3(v, w)$ for all v, w ; if $Y_3(v, w) = 0$, then $X_4(v, w) = X_3(v, w) = \mathbf{0}$; if $Y_3(v, w) = j > 0$, then $X_{4j'}^b(v, w) = 0$ for $j' \neq j$ and $X_{4j}^b(v, w) = b_j^w$.

The new mechanism, M_4 , satisfies condition 4 by construction and it inherits the other three conditions and acceptability from M_3 . Because it retains budget-balance and the same outcome function as M_3 , it has the same expected efficiency.

Let $\hat{M} = M_4$. We have shown that \hat{M} is acceptable and has expected efficiency at least as great as M . Because it satisfies conditions 1 through 4, it is also separable. \square

Proof of Lemma 4.

Proof. Suppose $M = (Y, X)$ is a separable, acceptable mechanism that does not feature a reserve. For each v , define $r(v) = \inf\{b | Y^s(v, b) = 1\}$. SF implies that if $Y^s(v, b) = 1$, $b \geq \sum_i X^s(v, b)$ for all v, b . Under PK, FRP implies that $X^s(v, b) \geq v_i^*$, which means that $\sum_i X^s(v, b) \geq \sum_i v_i^*$. Thus, $Y^s(v, b) = 1 \implies b \geq \sum_i v_i^*$, meaning that efficiency is a necessary condition for sale.

Because the mechanism does not feature a reserve, for some v , there exists b and b' , with $b' > b$, such that $Y^s(v, b) = 1$, but $Y^s(v, b') = 0$. However, because sale is efficient at b , it must also be efficient at b' , i.e. $b' > b \geq \sum_i v_i^*$.

Define a new mechanism, $\hat{M} = (\hat{Y}^s, \hat{X}^s)$, as follows. For any v, b , if $b < r(v)$, $\hat{Y}^s(v, b) = Y^s(v, b)$ and $\hat{X}^s(v, b) = X^s(v, b)$; if $b \geq r(v)$, then $\hat{Y}^s(v, b) = 1$ and $\hat{X}^s(v, b) = X^s(v, b)$ whenever $Y^s(v, b) = 1$ and $\hat{X}_i^s(v, b) = X_i^s(v, r(v)) + \frac{1}{n}[b - r(v)]$ whenever $Y^s(v, b) = 0$.

The new mechanism, \hat{M} , clearly has greater expected efficiency than M because $\hat{Y}^s(v, b) = 0 \implies Y^s(v, b) = 0$ and $\hat{Y}^s(v, b) > Y^s(v, b) \implies b > \sum v_i^*$. \square

Proof of Proposition 2.

Proof. FRP implies that $\alpha_i(v_{-i}, b)b \geq v_i$, so for any approved sale, $b \geq \frac{v_i}{\alpha_i(v_{-i}, b)}$. This must be true for all i , so $r(v) \geq \max_i \left\{ \frac{v_i}{\alpha_i(v_{-i}, b)} \right\}$.

Next, suppose that for some b, b' , $b \neq b'$ and $\alpha_i(v_{-i}, b) \neq \alpha_i(v_{-i}, b')$. Under PK, we cannot rule out the possibility that buyer j believes that by announcing $w_j \neq w_j^*$ he will increase the probability of a profitable sale by manipulating seller i 's share and, consequently, $r(v)$. Thus,

to preserve IC for the buyers under PK, it must be the case that $\alpha_i(v_{-i}, b)$ is independent of b .

Similarly, we cannot rule out the possibility that seller $i' \neq i$ would try to increase the probability of sale by manipulating seller i 's share and thus the auction reserve. Thus, to preserve IC for the sellers under PK, $\alpha_i(v_{-i}, b)$ must also be independent of v_{-i} . This means that for each i there exists some $\alpha_i \in [0, 1]$ such that $\alpha(v_{-i}, b) = \alpha_i$ for all $v_{-i} \in \mathbf{R}^{n-1}$ and $b \in \mathbf{R}$.

Finally, under PK, for each seller i we cannot rule out the possibility that $v_i^* > 0$ and that i believes that there is a positive probability of sale if she announces $v_i = v_i^*$. For such an i , if $\alpha_i = 0$ then i will believe that she can increase her expected payoff by increasing her announced value enough to lower the probability of sale, in violation of IC. Thus, $\alpha_i > 0$ for all i . □